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Optimization of the Dow Jones industrial average using the Treynor-Black model with appraisal weighting

Optimización ponderada por appraisal del índice Dow Jones industrial average mediante el modelo Treynor-Black

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Abstract

Objective: To evaluate whether historical performance (α) allows the dynamic weighting of the Dow Jones Industrial Average components to be optimized using the Treynor–Black model and to improve performance as related to the index.
Methodology: For the assets that compose the DJIA in each period, α is estimated using CAPM and multifactor models to construct an active portfolio weighted by the appraisal ratio (α/idiosyncratic variance), applying trailing rolling windows and no leverage. The paper also adds a chronological 80%-20% temporal validation and a progressive Jensen alpha estimated with a 252-day rolling window.
Results: The CAPM-based model achieves an average annual return close to 13.8% compared to 7.8% for the index, with an approximate 66.9% probability of outperforming it and a superior risk-return relationship. The 80%-20% temporary validation preserves the advantage out of sample: in the OOS block, the portfolio records a mean return of 13.7% versus 10.3% for the DJIA and an average progressive Jensen alpha of 0.0470.
Limitations: The results depend on the stability of α, changes in market regimes, sensitivity in the estimation of specific risk, and implementation costs that are not modeled in the validation exercise.
Originality: It integrates performance-based allocation (appraisal), preserving temporary comparability with the DJIA.
Conclusions: Appraisal-based weighting improves relative performance when the signals are statistically robust.

Keywords: Fintech acquisitions; financial performance; difference-in-differences; subgroup analysis; mergers and acquisitions.
JEL Classification: G34, G21, L86, C23, M10.

Resumen

Objetivo: Evaluar si el desempeño histórico (α) permite optimizar la ponderación dinámica de los componentes del Dow Jones Industrial Average (DJIA)mediante el modelo Treynor-Black y mejorar el desempeño frente al índice.
Metodología: Para los activos que integran el DJIA en cada periodo, se estima α con CAPM y modelos multifactoriales para construir un portafolio activo ponderado por el ratio appraisal (α/varianza idiosincrática) mediante ventanas móviles retrospectivas y sin apalancamiento. Se incorpora una validación temporal cronológica 80%-20% y un alpha de Jensen progresivo con ventana móvil de 252 días.
Resultados: El modelo basado en CAPM alcanza un rendimiento promedio anual cercano al 13.8% frente a 7.8% del índice, con una probabilidad aproximada de 66.9% de superarlo y mejor relación riESGo-rendimiento. La validación temporal 80%-20% mantiene la ventaja fuera de muestra: en el bloque OOS, el portafolio registra una media de 13.7% frente a 10.3% del DJIA y un alpha de Jensen progresivo promedio de 0.0470.
Limitaciones: Los resultados dependen de la estabilidad de α de cambios en régimen de mercado y de la sensibilidad en la estimación del riESGo específico.
Originalidad: Integra la asignación por desempeño (*appraisal*), preservando comparabilidad temporal con el DJIA.
Conclusiones: La ponderación basada por desempeño (appraisal) mejora el desempeño relativo cuando las señales son estadísticamente robustas.

Palabras clave: Adquisiciones Fintech; desempeño financiero; diferencias en diferencias; análisis de subgrupos; fusiones y adquisiciones.
Clasificación JEL: G34, G21, L86, C23, M10.

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Introduction

This study examines whether the components of the Dow Jones Industrial Average (DJIA) can be dynamically weighted to improve the index's risk-adjusted performance. The paper applies the Treynor-Black (TB) model, which separates a passive market block from an active security-selection block. In the passive block, the DJIA represents the benchmark exposure. In the active block, each DJIA component is evaluated using its estimated alpha and idiosyncratic risk; active positions are then weighted by the appraisal ratio, that is, alpha per unit of residual risk (Treynor and Black, 1973).

The advantage of Treynor-Black over a traditional index, equal-weight, or unconstrained mean-variance allocation is that it does not treat all securities as equally informative, nor does it rely only on total expected return. Instead, it converts security analysis into a disciplined active overlay: systematic risk remains in the benchmark block, while only statistically supported abnormal performance is allowed to influence the active block. This structure is especially useful for a bounded universe such as the DJIA because it allows a direct comparison with the index, limits active bets through concentration and no-leverage constraints, and makes the source of value -alpha, residual risk, and the passive-active mix- auditable.

Research framework and contribution. The empirical framework follows a chronological sequence: (i) define the DJIA constituents available at each formation date; (ii) estimate alpha, beta, and residual risk using only trailing information available up to that date; (iii) construct the active portfolio using appraisal-weighted positions subject to the stated restrictions; (iv) evaluate realized performance over the subsequent 252 trading days; and (v) validate the persistence of the results with an 80%-20% chronological split and a rolling-window Jensen alpha. The contribution to the literature is threefold. First, the paper adapts the TB rule to a changing but bounded DJIA universe, preserving comparability with the benchmark. Second, it evaluates whether appraisal weighting adds value beyond selecting securities only by positive alpha. Third, it documents the conditions under which the approach is applicable: the results are informative for large, liquid index universes, but should not be generalized to other markets without analogous out-of-sample validation, cost controls, and regime checks.

Active performance, signals, and risk measurement

The usefulness of the Treynor-Black (TB) model depends critically on the quality of the inputs: estimated alphas and idiosyncratic risks. Treynor and Black (1973) derive that the optimal weight of each active bet is proportional to its appraisal ratio, but the estimation of alphas tends to be the sensitive part of the model. Busse and Irvine (2006) demonstrate that a Bayesian approach that reduces alphas to zero when evidence is weak improves the detection of persistent skill; thus, the active block does not overweight and avoids model overfitting. Jones and Shanken (2005) incorporate learning between funds: investors update their beliefs not only based on a manager's performance, but also on the performance of their peers, which induces persistence and flows that can be confused with skill. The lesson for DJIA-Treynor-Black is not to mechanically read recent performance as stable alpha. Hunter et al., (2013) propose an Active Peer Benchmark that captures the common component among active strategies within the same cohort; discounting that group average when estimating alphas reduces the risk of paying for shared exposures rather than idiosyncratic skill. Similarly, Harvey and Liu (2021) warn about the "zoo of factors": with hundreds of tested signals, many "work" by chance if not corrected for multiple testing. This warning is central to Treynor-Black: an appraisal ratio inflated by data snooping pushes weight toward spurious signals. Benhamou and Guez (2021) provide a toolbox for breaking down the marginal contribution of each asset to performance ratios such as Sharpe, Sortino, or even maximum drawdown; in practice, this allows the DJIA manager to prioritize cuts or increases where the return per unit of specific risk is highest. Taken together, this literature calls for robustness checks, multiplicity corrections, structured learning, and marginal attribution before setting TB weights (Treynor and Black, 1973; Busse and Irvine, 2006; Jones and Shanken, 2005; Hunter et al., 2013; Harvey and Liu, 2021; He, 2007; Benhamou and Guez, 2021; Lo, 2008).

Signals by action, factors, and diversification

The active Treynor-Black block on the DJIA can be fed by signals per-share (bottom-up) or exposures to aggregate factors (top-down). Heinrich, et al., (2021) show that, in long-only portfolios on the S&P 500 and STOXX 600, incorporating an efficient covariance estimator to diversify specific risk improves performance compared to concentrating all weight on the stocks with the highest signal. In long-short, the differences are reduced, but the central message remains: diversifying signals reduces idiosyncratic risks without overly diluting useful "information" (Heinrich et al., 2021).

Additionally, Zurek and Heinrich (2020) compare bottom-up and top-down approaches from an alpha forecasting perspective: when alpha concentration is high, stock-picking architectures would dominate; when signals are weak or highly correlated, top-down or hybrid approaches prove more stable. Ross (2021) documents that interactions between characteristics—for example, size and valuation—provide robust incremental information for predicting returns; incorporating interaction terms can raise the appraisal ratio of active bets on the DJIA compared to using simple additive effects (Ross, 2021). On the product front, Nuorlahti (2021) analyzes smart beta ETFs in the US and finds low relative returns overall, with partial improvements in bearish scenarios; the result suggests that "packaging" factors does not guarantee net alpha after costs, a direct warning not to confuse exposure to styles with selection ability in TB. In emerging markets, Ali et al., (2019) find mixed evidence for the five-factor model: some premiums appear unstable by period, reinforcing the need to prove the validity of signals locally before using them as alphas in TB. For their part, De Roon, Nijman, and Ter Horst et al., (2000) criticize returns-based style analysis when rigid restrictions are imposed; poor specification of "styles" can skew alphas and, with them, TB weights. This converges on a guide for DJIA-Treynor-Black: i) prefer signals per stock when they are informative, ii) diversify the active block with stable covariance matrices, iii) model interactions between characteristics, iv) distinguish exposure to true alpha factors, and v) avoid "style" measurements that contaminate the appraisal ratio (Heinrich et al., 2021; Zurek and Heinrich, 2020; Ross, 2021; Nuorlahti, 2021; Ali et al., 2021; De Roon et al., 2004).

Non-standard risk, fall prevention systems, and management

The Treynor-Black weighting formula uses specific variance as a measure of "noise." However, in real markets, the tails are thick and regimes change. Reuss et al., (2016) α-stable distributions and switching regimes to capture asymmetries and variable volatility; when $\alpha < 2$, variance may not even exist, so they recommend convex measures such as CVaR instead of VaR. Translated to DJIA-Treynor-Black, estimating idiosyncratic risk with robust scales or CVaR reduces sensitivity to extreme events that would inflate or underestimate residual risk (Reuss et al., 2016).

From a multi-period perspective, Jarvis et al., (2009) show that dynamic policies that optimize

TailVaR outperform single-period schemes in meeting maximum loss targets, while Infanger (2006) uses stochastic dynamic programming to demonstrate that preferences and horizon matter in allocation, beyond static Sharpe. These ideas suggest that the mix between the passive DJIA block and the active TB block can be adjusted over time to respect downside limits or expected welfare. In operational hedging terms, Perold and Sharpe (1988) compare dynamic rules such as constant-mix and CPPI: the former buys on dips and sells on rallies, while the latter does the opposite. Applied to DJIA-TB, a CPPI profile enhances the contribution of the active block when the signal is strong, while a constant-mix dampens cyclicality, modulating the aggregate risk-return relationship (Perold and Sharpe, 1988). At the same time, Harvey and Liu (2021) point out that many reported "risk signals" may be the result of multiple testing; adopting robust risk measures does not exempt one from statistically validating the persistence of alphas. Taken together, this literature pushes for a renewal of the "R" component of the appraisal ratio in TB: replacing variance with robust measures in tails, recognizing regimes, and choosing passive-active blending policies consistent with tolerable losses, not just averages (Reuss et al., 2016; Jarvis et al., 2009; Infanger, 2006; Perold and Sharpe, 1988; Harvey and Liu, 2021).

Applied evidence: hedge funds and investment managers

The direct application of Treynor-Black in real contexts warns of friction, costs, and database biases. Pannu (2021) implements TB in a fund-of-funds of hedge funds, forecasts information ratios by strategy, and compares with a 1/N allocation; TB outperforms in part of the period, underperforms in others, and sensitivity to costs and regime changes is high. Manap et al., (2024) evaluate investment companies using the TB method, reporting improvements in Sharpe and Treynor ratios and positive Jensen alphas, but warn of dependence on historical estimates, weight sensitivity, and data quality. Rezaei and Nezamabadi-Pour (2025) test deep reinforcement learning algorithms (A2C, PPO, SAC, TD3, among others) on a universe of 30 DJIA stocks and show that they can improve performance metrics without explicitly predicting prices; these DRL signals are candidates for alphas for the active TB block if validated outside the sample. Venturato (2018) shows how a Kalman filter on Fama-French factors generates "smoothed" alphas that are more stable for TB, although their advantage is reduced when frictions are incorporated. Structurally, Kurtti (2020) discusses "how many stocks are enough" to diversify: there is no universal number, but the optimum results from the balance between non-systematic risk reduction and complexity; in TB, this supports an active block with few high-conviction bets within the DJIA. At the fund level, Cuthbertson et al., (2010) review measurements and evidence and find that the average net alphas of active funds are weak after costs, reinforcing that the active TB layer must control turnover and frictions. In emerging markets, Salardini et al., (2020) find that the preference for active funds over indexes is not explained by higher net returns, but rather by index tracking errors and behavioral factors. The lesson for the DJIA-TB is that the passive block must accurately replicate the index so as not to "erode" the value of the active block. In thematic investing, Berg et al., (2023) document that adding ESG ratings from multiple agencies and weighting them in TB style enhances the performance of ESG portfolios, suggesting that thematic signals should be integrated into the active layer of the DJIA with TB rules. Finally, as a didactic basis, Ceballos Bejarano et al., (2025) implement Markowitz by quadratic programming to construct efficient frontiers, a direct input for the passive block and to stabilize covariances of the active block in TB. The set draws an applied map: signals predicted by learning or filters, active blocks with few bets, friction control, and accurate replication of the DJIA in the passive block (Arisena et al., 2018; Pannu, 2021; Manap et al., 2024; Rezaei and Nezamabadi-Pour, 2025; Venturato, 2018; Kurtti, 2020; Cuthbertson et al., 2010; Salardini et al., 2020; Berg et al., 2023; Ceballos Bejarano et al., 2025). **Table 1** summarises the main approaches and conclusions.

Divergences, and gaps: where the literature converges and what remains to be resolved

There are four clear convergences. First, the appraisal ratio is the value lever of the TB active block, but only if the alpha is genuine and the residual risk is well measured (Treynor and Black, 1973; French, 2003; Benhamou and Guez, 2021). Second, active block diversification matters: too much concentration overloads idiosyncratic risk, while diversification with stable covariances preserves "information" without diluting it (Heinrich et al., 2021). Third, validating signals and discounting common modes or multiplicity reduces overfitting (Busse and Irvine, 2006; Hunter et al., 2013; Harvey and Liu, 2021). Fourth, dynamic rules and robust measures to tails/regimes improve alignment with loss and welfare objectives (Reuss et al., 2016; Jarvis et al., 2009; Infanger, 2006; Perold and Sharpe, 1988). Among divergences, the literature differs on signal architecture: bottom-up dominates with high alpha concentration, but top-down or hybrids stabilize when signals are correlated or weak (Zurek and Heinrich, 2020). Likewise, evidence on smart beta products suggests caution: exposure to factors does not equate to generating net alpha (Nuorlahti, 2021), while in emerging markets the validity of premiums is not always sustained (Ali et al., 2019).

In practice, TB works but is sensitive to costs, regime, and data quality (Pannu, 2021; Manap et al., 2024), which requires realistic frictions in construction. Several gaps remain. There is a lack of a unified framework that integrates DRL, Kalman, and feature interactions with shrinkage filters and multiplicity corrections, such as a stack of models that maximizes the appraisal ratio subject to CVaR under multiple regimes (Rezaei and Nezamabadi-Pour, 2025; Venturato, 2018; Ross, 2021; Busse and Irvine, 2006; Harvey and Liu, 2021; Reuss et al., 2016; Vargas, 2012). There is also a lack of systematic evaluation of the trade-off between "few high-conviction bets" and "diversification of the active block" in the DJIA with realistic costs, extending the intuition of Kurtti (2020) and Heinrich et al., (2021). Finally, there is an urgent need to normalize value attribution across blocks and regimes, combining Lo's (2008) decomposition with Active Peer Benchmark to avoid paying for peer "fads" in the DJIA (Hunter et al., 2013; Lo, 2008).

Principles for using the TB model

The literature on the subject agrees that the Treynor-Black model is an efficient framework for converting selection signals into weighting decisions on a broad index, such as the DJIA, provided that three principles are respected. First, robust alphas: they must arise from models that integrate learning and economic structure, undergo shrinkage, correct for multiplicity and discount peer fads, and consider interactions between characteristics (Treynor and Black, 1973; Busse and Irvine, 2006; Harvey and Liu, 2021; Hunter et al., 2013; Ross, 2021). Second, well-measured idiosyncratic risk: stable covariance matrices and, under tails/regimes, robust measures such as CVaR are crucial to avoid distorting the denominator of the appraisal ratio (Heinrich et al., 2021; Reuss et al., 2016). Third, blending with the DJIA guided by tail objectives and frictions: dynamic CPPI or constant-mix rules help tune active exposure based on the environment and loss limits, and index replication must be accurate so as not to "eat up" alpha (Perold and Sharpe, 1988; Jarvis et al., 2009; Infanger, 2006;

Salardini et al., 2020). In modern applications, DRL and Kalman signals enable actionable alphas if rigorously validated, and TB extends to themes such as ESG through rating aggregation and alpha/noise weighting (Rezaei and Nezamabadi-Pour, 2025; Venturato, 2018; Berg et al., 2023).

Methodology

The methodology of this study is designed to address the challenges of portfolio diversification and active management, with a particular focus on the role of asset weighting and the use of the Treynor-Black model. Below, we provide a more detailed explanation of the key aspects of the methodology, including the importance of asset weighting, the treatment of potential biases in the dataset, the rationale for limiting portfolios to 30 assets, and a deeper discussion of the appraisal ratio equation (9), which serves as the core of the study. The reason why the DJIA was chosen is because access to the index components during the study period is available. These components may change each year, and this is taken into account in the study. By using the same components, the study is not influenced by asset selection (stocks/components).

Portfolio universe, data treatment, and weighting constraints

Asset weighting plays a critical role in portfolio diversification. Properly weighted portfolios can reduce unsystematic risk while maximizing returns. However, increasing the weighting of certain assets in an active portfolio can also lead to higher risk, particularly if the portfolio manager's ability to select assets is not sufficiently robust (Brands et al., 2005). To mitigate this risk, we impose specific constraints on portfolio construction. For example, we avoid leveraged portfolios and set a maximum weight for individual assets to prevent over-concentration in any single asset. These constraints ensure that the portfolio remains diversified and aligned with the investor's risk tolerance.

Treatment of potential biases in the dataset

One of the challenges in portfolio management is addressing potential biases in the dataset. In this study, we acknowledge that no specific treatment was applied to correct for biases in the data. This decision was made to avoid the loss of information that could result from nonlinear data treatments. By using the raw data, we preserve the integrity of the dataset and ensure that our analysis reflects the true behavior of the assets under study. While this approach may introduce some bias, it allows for a more transparent and interpretable analysis.

Rationale for limiting portfolios to 30 assets

The portfolios in this study are composed of a maximum of 30 assets, corresponding to the number of components in the Dow Jones Industrial Average (DJIA). The DJIA was chosen as the focus of this study because it provides a well-defined and widely recognized set of assets, making it an ideal benchmark for analyzing the impact of asset weighting on portfolio performance. By limiting the portfolios to 30 assets, we ensure that our analysis is directly applicable to the DJIA and avoids the complexities associated with larger or more diverse asset universes. This approach allows us to concentrate on the role of weighting rather than asset selection.

Active investment management can be evaluated with alpha (α), beta (β), volatility, tracking error, Sharpe ratio, information ratio, and appraisal ratio, among other measures (Lo, 2008). This paper follows Treynor and Black (1973), for whom appraisal is the core criterion for moving from security analysis to portfolio weights. The portfolio has two components: a passive portfolio that represents the benchmark and an active portfolio that contains securities with positive and statistically significant alpha.

The active weight for security i at formation date t is proportional to its appraisal score:

$$\omega_i = \frac{\alpha_i / \sigma_i^2}{\sum_{j=1}^{N} \alpha_j / \sigma_j^2} \tag{1}$$

The proportion in the active portfolio is determined with equation 6 (for more details see Treynor & Black, 1973).

$$\alpha_A = \sum \omega_i \alpha_i \tag{2}$$

$$\beta_A = \sum \omega_i \beta_i \tag{3}$$

$$\sigma_A^2 = \sum \omega_i^2 \sigma_i^2 \tag{4}$$

$$\omega_0 = \frac{\alpha_A / \sigma_A^2}{(R_M - R_F)/\sigma_M^2} \tag{5}$$

$$\omega_A = \frac{\omega_0}{1 + (1 - \beta_A)\omega_0} \tag{6}$$

For the calculation of alpha in equation 1, the five-factor model of Fama & French (2015) is used, see equation 7.

$$R_i - R_f = \alpha_i + b_i(R_i - R_f) + s_i SMB_i + h_i HML_i + r_i RMW_i + c_i CMA_i + \varepsilon_i \tag{7}$$

Where $R_i - R_f$ is the market premium, α_i is the performance, the coefficients b_i, s_i, h_i, r_i, c_i are obtained by linear regression and SMB_i, HML_i, RMW_i, CMA_i are the portfolios that mimic the size, book-to-market, profitability and investment factors, respectively. ε_i is the error term of the model. These factors include all NYSE, AMEX, and NASDAQ stocks. For more detail, see appendix: "Explanation of the Fama-French Five-Factor Model".

Yield is different from performance. Yield is the division of two values, profit by initial investment. While performance includes all prices in the investment horizon. Which are adjusted based on the risk relationship with a benchmark (DJIA). The returns are then adjusted for the assumed risk using equation (8). This adjustment is made with the CAPM equation (8), with alpha being the abnormal return or additional return that corresponds due to the risk taken compared to the market risk. If it is positive, it means that the strategy pays a better return due to the risk taken compared to the market.

$$(r_i - r_f) = \alpha + \beta(r_{DJIA} - r_f) + \varepsilon \tag{8}$$

This abnormal return is due to the strategy's own risk (unsystematic risk), which does not consider market risk (systematic risk). To compare alphas, the relationship with the unsystematic risk is calculated equation (9). The higher the appraisal, the higher the risk-return ratio of the strategy. The appraisal of the benchmark is assumed to be zero since it has an alpha of zero (see Amenc & Le Sourd; 2003).

$$Appraisal = \frac{\alpha}{var[\varepsilon]} = \frac{\alpha}{\sigma_\varepsilon^2} \tag{9}$$

α is the excess return of the portfolio relative to the benchmark (DJIA).

α_ε^2 is the standard deviation of the portfolio's unsystematic risk.

The appraisal ratio, defined in equation (9), is a central metric in this study. It measures the risk-adjusted performance of a portfolio by comparing the excess return (alpha) to the unsystematic risk of the portfolio. It provides a measure of the portfolio's performance relative to the risk taken. A higher appraisal ratio indicates a more favorable risk-return profile, as the portfolio generates greater excess returns for a given level of unsystematic risk. This metric is particularly useful for evaluating active portfolios, as it captures the manager's ability to generate alpha while controlling for risk.

In practical terms, the appraisal ratio serves as the basis for determining the weights of assets in the active portfolio. Assets with higher appraisal ratios are given greater weight, as they contribute more to the portfolio's risk-adjusted performance. This approach aligns with the Treynor-Black model, which combines a passive portfolio (the benchmark) with an active portfolio (selected assets) to achieve superior performance.

In practical terms, the appraisal ratio serves as the basis for determining the weights of assets in the active portfolio. Assets with higher appraisal ratios are given greater weight, as they contribute more to the portfolio's risk-adjusted performance. This approach aligns with the Treynor-Black model, which combines a passive portfolio (the benchmark) with an active portfolio (selected assets) to achieve superior performance. Importantly, the appraisal ratio used to set weights at date t is calculated from the same trailing information set used to estimate alpha and residual risk; future realized returns are not used to rank assets or determine weights.

Chronological estimation protocol. To avoid look-ahead bias, the empirical implementation follows a strict sequence for each portfolio-formation date t: (i) estimate alpha, residual variance, and appraisal using only a trailing rolling window ending at t; (ii) form the active portfolio weights at t using those estimates; and (iii) evaluate realized performance over the subsequent 252 trading days. The realized return from $t+1$ to $t+252$ is therefore an evaluation target, not an input in the estimation or weighting process.

Temporary 80%-20% validation protocol. To address the practical-applicability concern associated with forward-looking estimates, the validation panel is split chronologically into an 80% in-sample block and a 20% out-of-sample block. The full validation panel contains 5,134 trading days; the split is placed at row 4,107, so the in-sample period covers rows 1-4,107 and the out-of-sample period covers rows 4,108-5,134. The trading rules, including the $R^2 >= 0.70$ and p-value $<= 0.05$ filters, are not re-estimated in the OOS block.

The progressive Jensen alpha is estimated at each trading day k by regressing portfolio excess returns over the risk-free rate on DJIA excess returns over the risk-free rate, using only the previous 252 trading days. Therefore, alpha at k uses information available up to k and is not the intercept from a regression over the entire sample. The full-sample CAPM intercept is reported only as a retrospective benchmark and is not used as an operational alpha. All validation returns are gross of transaction costs; the direct extension is to subtract a cost proportional to absolute turnover before computing the validation statistics.

Figure 1 summarizes the performance of assets that were at some point part of the DJIA. **Figures 2** and **3** show the entire research methodology.

Relation with the literature and external validity

The comparison with applied studies shows agreement on three fronts. First, when alphas are generated by models that integrate economic structure and learning, the active layer produces consistent improvements over the index, although the advantage is reduced with costs and in adverse regimes (Pannu, 2021; Rezaei and Nezamabadi-Pour, 2025). Second, Kalman filters that "smooth" factor coefficients reduce weight rotation and stabilize the active contribution, which aligns the signal with institutional investors' decision horizons (Venturato, 2018). Third, adding signals from various sources and weighting them in the TB style increases robustness, as documented in ESG thematic portfolios; the operational parallel with DJIA-TB is to treat each signal provider as an "analyst" and combine evidence with a rule of weights per appraisal (Berg et al., 2023). By imposing weight limits and avoiding leverage, the tables in our study offer a prudent design that reduces tracking error and facilitates adoption by mandates with risk constraints. This approach also addresses a classic criticism: gross alphas may not survive transaction costs and fees; evidence in active funds suggests weak net advantages, so any innovation must be parsimonious in rotation and frictions (Cuthbertson et al.,, 2010; Salardini et al.,, 2020). In this sense, the novel contribution lies in integrating the TB rule with a bounded universe, concentration limits, and robust residual risk metrics, maintaining a blending plan with the index that respects loss thresholds and is auditable with window backtests that reflect regime shifts.

Limitations, implications, and external validity

The universe of 30 stocks facilitates estimation and risk control but restricts substitution when several signals fall simultaneously. In addition, the composition of the DJIA changes over time, and these rotations can affect the continuity of series. Returns by moving windows capture part of this effect, but not all of it. The decision not to perform bias treatments preserves information, although it leaves noise that can inflate the specific variance of certain securities; a reasonable compromise is to apply shrinkage to alphas and covariances to attenuate extremes without excessive "cleaning." The absence of leverage reduces the probability of large declines but also limits the upper bound of risk-adjusted return; the use of dynamic blending rules helps modulate this compromise without changing the risk mandate. Regarding external validity, dispersion patterns and active block effectiveness depend on the regime: in systemic crises, the marginal gain of the TB contracts; in more diversified environments, it reappears. These contingencies have already been reported in the literature on dynamic allocation and queue control (Jarvis, et al.,, 2009; Reuss, et al., 2016; Perold and Sharpe, 1988). Operationally, the results imply that an institutional implementation should maintain and report: i) out-of-sample validation with temporary purging and multiplicity control for alphas, ii) robust residual risk estimators, and iii) concentration limits with periodic review. For financial innovation, the study shows the viability of a minimum active layer, governed by appraisal, which coexists with the DJIA and can incorporate signals -e.g., reinforcement learning- without altering classic risk governance and tracking error (Rezaei and Nezamabadi-Pour, 2025; Lo, 2008; French, 2003; Treynor and Black, 1973).

Results

Table 2 (column "Alpha source") shows different models for calculating alpha. The best model is the CAPM where the market is the DJIA and alpha is only considered when the model has $R^2 >= 0.70$ and the alpha coefficient has a p-value $<= 0.05$ 0.05. In each simulation, alpha is estimated using only trailing information available up to the portfolio-formation date. With these alphas for the 30 components of the DJIA, the appraisal is calculated for each strategy and the portfolio is then evaluated over the subsequent 252 trading days. The CAPM-based model achieves an average return of 13.8% versus 7.8% for the DJIA and outperforms the DJIA in 66.9% of simulations. Although there are periods in which the portfolio is invested in only one asset, the risk-return ratio is better for the portfolio, with 0.821 versus 0.509 for the market (DJIA).

To contrast the contribution of alpha in the portfolio construction, we calculate the probability that an asset selected at date t (the only information available) up to t, outperforms the DJIA over the subsequent year or generates a return greater than zero. **Table 3** shows that, for the best model (second row), pure alpha selection has a 55.4% probability of outperforming the DJIA, compared with 66.9% for the appraisal-weighted portfolio in **Table 2** using the same model. This suggests that diversification through the Treynor-Black weighting rule contributes to a higher probability of outperformance than selection based only on positive alpha.

Table 5 reports the temporary 80%-20% validation. In the in-sample block, the portfolio obtains an average return of 8.09% compared with 3.92% for the DJIA. In the out-of-sample block, the portfolio also remains above the benchmark, with 13.73% compared with 10.33% for the DJIA. The out-of-sample evidence therefore supports the practical interpretation of the strategy: the advantage is not limited to the first 80% of the chronological panel.

The hit-rate evidence qualifies this result. The portfolio beats the DJIA in 45.29% of in-sample observations and 42.65% of out-of-sample observations. Thus, the strategy does not dominate by winning more often in the validation split; rather, its average advantage is generated by the magnitude of favorable periods when the appraisal-weighted allocation succeeds. This distinction is important because it separates the economic size of excess performance from the frequency of outperformance.

Figure 5 complements this evidence with the progressive Jensen alpha. The average rolling alpha increases from 0.0261 in the in-sample block to 0.0470 in the out-of-sample block, while the full-sample CAPM intercept is 0.0474. Since the rolling alpha is estimated using only past information within each 252-day window, the similarity between the OOS rolling alpha and the retrospective full-sample intercept provides a robustness check without treating the full-sample intercept as a tradable signal. The path of alpha is time-varying, which is consistent with regime-dependent active-management value and reinforces the need for rolling validation rather than a single static estimate.

Tables 6 and 7 show that the active portfolio, under the stated restrictions, outperforms other asset-allocation alternatives reported in the literature. Table 6 and Figure 4 indicate positive abnormal performance under the CAPM specification, while Table 7 reports a higher appraisal ratio, indicating a stronger relationship between performance and unsystematic risk. Table 8 shows the assets in this study.

Conclusion

This study evaluates whether the Treynor-Black model can improve the dynamic weighting of the Dow Jones Industrial Average components by using statistically significant alpha and idiosyncratic risk. The evidence indicates that appraisal-weighted active allocation improves performance relative to the DJIA when the signals are estimated chronologically and subject to prudent restrictions. In the best-performing CAPM specification, the active portfolio obtains an average annual return close to 13.8% compared with 7.8% for the index, and outperforms the DJIA in approximately 66.9% of the simulations. The improvement is not only a consequence of selecting positive alphas: pure alpha selection reaches about 55.4% outperformance, while appraisal weighting increases that probability by combining alpha with residual risk. The 80%-20% validation adds out-of-sample evidence: in the OOS block, the active portfolio reaches a mean return of 13.73% versus 10.33% for the DJIA, with an average progressive Jensen alpha of 0.0470.

The main contribution is to show how the TB rule can be operationalized in a small, transparent, and changing index universe. The DJIA serves as a passive benchmark, while the active layer is built from the same constituents available at each date. This design reduces asset-selection bias, maintains comparability with the index, and makes the weighting decision auditable. The chronological protocol is central to the interpretation of the results: alpha, residual variance, appraisal, and portfolio weights are estimated using information available only up to the formation date, whereas the following 252 trading days are used only for ex-post evaluation. The additional 80%-20% split further indicates that the strategy retains positive average performance outside the initial chronological block, even though the OOS hit rate remains below 50%.

The findings also define the limits of the analysis. The results should not be read as universal evidence that TB will dominate in every market. The DJIA universe is liquid and concentrated, and its composition changes through time. In periods of high market correlation or regime stress, the marginal value of active selection may decline. Moreover, implementation costs, turnover, and parameter uncertainty may reduce realized gains. The validation split does not eliminate all design-selection bias if model rules were chosen after inspecting the whole sample; instead, it reduces the forward-looking concern by separating the chronological evaluation window and by reporting alpha estimates obtained only from rolling historical information. For this reason, the model is most defensible when signals are statistically robust, leverage is avoided, concentration limits are imposed, and results are tested outside the estimation window.

Future lines of research

Future work should extend the analysis in five directions: (i) incorporate shrinkage estimators and multiple-testing controls for alpha; (ii) compare variance with tail-sensitive residual-risk measures such as CVaR; (iii) estimate the effect of transaction costs and turnover directly in the 80%-20% validation; (iv) evaluate dynamic passive-active mixing rules under explicit tracking-error or loss limits; and (v) repeat the temporary validation with alternative rolling-window lengths, purged cross-validation, and additional market regimes to test the predictive strength of the model beyond the DJIA sample.

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Table 1

Summary of principal literature

Author (year)	Universe/Data	Central focus	Key contribution to DJIA-TB	Notable limitations
Treynor y Black (1973)	Theory with examples	Passive-active breakdown and appraisal ratio	Weight rule for the active block; optimal mix with index	Ignore costs and taxes
Perold y Sharpe (1988)	2-assets	Dynamic rules (CPPI, mix)	TB-DJIA mixture control levers	Stylized assumptions
French (2003)	Conceptual	Treynor as an operational metric	Prioritization by alpha/idiosyncratic noise	It implies parameter stability
Jarvis et al., (2009)	General	Dynamic policies and TailVaR	Adjust TB-DJIA mix to tail targets	Models can be fragile
Pannu (2021)	Hedge funds	TB for fund-of-funds	Sensitivity of TB to treatment regimen and costs	Self-reported data
Berg et al., (2023)	EE. UU./UE/JP	ESG + TB	Signal aggregation + TB improves performance	Heterogeneity by supplier
Manap et al., (2024)	Investment companies	TB with classic metrics	Operational guide and feasibility	Sample bias, rebalancing
Rezaei y Nezamabadi-Pour (2025)	DJIA 30	DRL para señales	DRL como generador de alfas TB	Generalization and costs
Ceballos Bejarano et al.,, (2025)	5 assets	Markowitz	Stable covariances for TB	Reduced horizon and size

Source: own elaboration.

Note: TB = Treynor-Black; DRL = aprendizaje por refuerzo profundo; VaR = Valor-en-riESGo.

Table 2
Treynor-Black active portfolio: a comparison using different alpha (α) types, 2000-2020

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Alpha source	DJIA average annual return	Portfolio average annual return	DJIA volatility	Portfoliovolatility	Portfolio max. number of assets used	Portfolio min. assets used	Probability of outperforming the DJIA	DJIA	Portfolio
	(a)	(b)	(c)	(d)				(a / c)	(b / d)
CAPM (DJIA as the market)	6.30%	6.90%	14.90%	14.10%	21	7	58.00%	0.42	0.492
CAPM (DJIA as the market, $R^2 \geq 0.70$)	7.80%	13.80%	15.30%	16.80%	18	1	66.90%	0.509	0.821
F&F (1 factor model)	6.20%	5.90%	15.10%	13.90%	28	1	56.60%	0.411	0.427
F&F (2 factor model)	6.20%	5.90%	15.10%	13.90%	28	1	56.90%	0.411	0.427
F&F (3 factor model)	6.20%	5.90%	15.10%	13.90%	28	1	56.90%	0.411	0.428
F&F (4 factor model)	6.20%	5.90%	15.10%	13.90%	28	1	56.80%	0.411	0.427
F&F (5 factor model)	6.20%	5.90%	15.10%	13.90%	28	1	56.70%	0.411	0.427
Best selected model (higher adjusted $R^2 \geq 0.70$)	6.30%	6.90%	14.90%	14.00%	21	6	57.30%	0.42	0.489
Best selected model (higher adjusted R^2 & $R^2 \geq 0.70$)	7.70%	13.00%	15.30%	16.00%	18	1	65.70%	0.508	0.809

Source: own elaboration and data from FactSet.
Note: On each day between 2000 and 2020, alpha is estimated for each of the 30 assets belonging to the DJIA at that date using only information available up to that day, through a trailing rolling window. Only alphas with p-value $<= 0.05$ 0.05 are considered. For each day, assets with positive and significant alpha are weighted according to their appraisal ratio. There may be observations without investment. Probability of outperforming the DJIA: the portfolio formed at date t is compared ex post with the DJIA over the subsequent 252 trading days. These subsequent returns are used only for evaluation and are not used to estimate alpha, rank assets, or set portfolio weights. 5,134 simulations in total. For the model used, see Ali et al., 2019; and Fama et al., 2015.

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Table 3
Pure *alpha* investing: 2000-2020

Alpha source	Probability of outperforming the DJIA	Probability of returns greater than zero	observations
CAPM (DJIA as the market)	50.20%	63.80%	70 162
CAPM (DJIA as the market, $R^2 \geq 0.70$)	55.40%	72.10%	15 565
F&F (1 factor model)	49.80%	62.20%	91 784
F&F (2 factor model)	49.80%	62.20%	91 735
F&F (3 factor model)	49.80%	62.20%	91 684
F&F (4 factor model)	49.80%	62.20%	91 671
F&F (5 factor model)	49.80%	62.20%	91 667
Best selected model (higher adjusted R^2)	50.30%	64.00%	70 016
Best selected model (higher adjusted R^2 & $R^2 \geq 0.70$)	55.10%	71.50%	15 967

Source: own elaboration.

Note: P-value $<= 0.05$. There may be observations without investment. Probability of outperforming the DJIA: each asset is selected at date t using only information available up to t, and its subsequent one-year performance is compared ex post with the DJIA. The subsequent 252-trading-day return is not used as an input to estimate alpha or select the asset. 5,134 simulations in total.

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Table 4
Treynor & Black (1973) model comparison: 2000-2020

Alpha source	DJIA average annual return	**Portfolio average annual return**	DJIA volatility	**Portfolio-volatility**	**Portfolio max. number of assets used**	**Portfolio min. assets used**	**Probability of outperforming the** DJIA	DJIA	**Portfolio**
	(a)	(b)	(c)	(d)				a / c	b / d
Active portfolio	7.80%	13.80%	15.30%	16.80%	18	1	66.90%	0.509	0.821
Active + Passive portfolio	7.80%	9.10%	15.30%	16.00%	18	1	24.60%	0.509	0.567

Source: own elaboration and data from FactSet.

Note: Probability of outperforming the DJIA: for each simulation, the portfolio weights are set at date t using only information available up to t, and the portfolio is then compared ex post with the DJIA over the subsequent 252 trading days. Because weights may vary across dates, each simulation can have a different asset allocation. 5,134 simulations. Both in terms of yield and risk-return ratio, the active portfolio outperforms the passive portfolio. When volatility is considered as risk, the mistake is made of including positive performance deviations as risk, when an increase in performance is not risk, it is volatility. This is corrected in **Table 4**.

Table 5
Temporal 80%-20% validation and progressive Jensen alpha: 2000-2020

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Partition	DJIA mean return	Portfolio mean return	Hit rate (rp > rm)	Mean progressive Jensen alpha
In-sample (80%)	3.920%	8.090%	45.29%	0.026129
Out-of-sample (20%)	10.325%	13.725%	42.65%	0.047000

Source: own elaboration and data from FactSet.

Note: The validation panel contains 5,134 trading days. The chronological split is placed at row 4,107 (IS: rows 1-4,107; OOS: rows 4,108-5,134). Returns are gross and do not include transaction costs. Progressive Jensen alpha is estimated with a 252-trading-day rolling CAPM using portfolio and DJIA excess returns over the risk-free rate. The full-sample retrospective CAPM intercept is 0.047368 and is reported only as a methodological reference, not as an operational signal.

Table 6
Performance of portfolios using the CAPM: 2000-2020

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	Factor	Standard error	T-statistic	P-value
Active + Passive portfolio (R-squared 0.868)				
Alpha	0.015	0.001	13.383	0.000
Beta	0.974	0.006	150.776	0.000
Active portfolio (R-squared 0.496)				
Alpha	0.078	0.002	34.045	0.000
Beta	0.773	0.013	58.104	0.000

Source: own elaboration and data from FactSet.

Note: *Alpha* indicates the potential-adjusted abnormal return. The CAPM adjusts returns based on risk. Alpha can be viewed as the excess return, in addition to the return due to the market/system/country/sector. Beta is the risk level of the portfolio, compared to the market. Where the market has a beta of 1. A beta less than 1 is a portfolio with lower risk. In summary, the active portfolio has better performance (higher alpha) and a risk close to the DJIA.

Table 7
Appraisal ratio for different passive portfolios: 2000-2020

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			Passive investments portfolios: optimization strategy					
	Mean-variance	Equal-weight	Downside-potential	Upside-potential	Semi-variance	Omega-ratio	Active + Passive portfolio	Active portfolio
Appraisal	$-\alpha$	$-\alpha$	1.64	1.13	1.34	1.74	4.43	5.47

Source: own elaboration and data from FactSet.

Note: Higher appraisal indicates better portfolio optimization. Active portfolio and Passive portfolio with Treynor & Black (1973). When comparing different ways to weight portfolios, the active portfolio has a better performance (best appraisal) compared to the rest.

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Table 8
Components in the history of the DJIA: 199Q4-2021Q2

	Company name	FactSet			Company name	FactSet
1	Alcoa Inc.	AA-US		30	JPMorgan Chase & Co.	JPM-US
2	Apple Inc.	AAPL-US		31	The Coca-Cola Company	KO-US
3	American International Group, Inc.	AIG-US		32	McDonald's Corporation	MCD-US
4	Amgen Inc.	AMGN-US		33	Kraft Foods Inc.	MDLZ-US
5	American Express Company	AXP-US		34	3M Company	MMM-US
6	The Boeing Company	BA-US		35	Minnesota Mining & Manufacturing Company	MMM-US
7	Bank of America Corporation	BAC-US		36	Altria Group Incorporated	MO-US
8	Citigroup Inc.	C-US		37	Philip Morris Companies Inc.	MO-US
9	Caterpillar Inc.	CAT-US		38	Merck & Co., Inc.	MRK-US
10	salesforce.com, inc.	CRM-US		39	Microsoft Corporation	MSFT-US
11	Cisco Systems, Inc.	CSCO-US		40	General Motors Corporation	MTLQQ-US
12	Chevron Corporation	CVX-US		41	Nike, Inc.	NKE-US
13	DowDuPont Inc.	DD-US		42	Pfizer Inc.	PFE-US
14	E.I. du Pont de Nemours & Company	DD.XX1-US		43	The Procter & Gamble Company	PG-US
15	The Walt Disney Company	DIS-US		44	Raytheon Technologies Corporation	RTX-US
16	Dow Inc.	DOW-US		45	United Technologies Corporation	RTX-US
17	Eastman Kodak Company	EKDKQ-US		46	AT&T Corporation	T.XX1-US
18	General Electric Company	GE-US		47	SBC Communications Inc.	T-US
19	The Goldman Sachs Group, Inc.	GS-US		48	AT&T Inc.	T-US
20	The Home Depot, Inc.	HD-US		49	The Travelers Companies, Inc.	TRV-US
21	AlliedSignal Incorporated	HON-US		50	UnitedHealth Group Inc.	UNH-US
22	Honeywell International	HON-US		51	Visa Inc.	V-US
23	Honeywell International Inc.	HON-US		52	Verizon Communications Inc.	VZ-US
24	Hewlett-Packard Company	HPQ-US		53	Walgreens Boots Alliance, Inc.	WBA-US
25	International Business Machines Corporation	IBM-US		54	Walmart Inc.	WMT-US
26	Intel Corporation	INTC-US		55	Wal-Mart Stores, Inc.	WMT-US
27	International Paper Company	IP-US		56	Exxon Mobil Corporation	XOM-US
28	Johnson & Johnson	JNJ-US		57	Exxon Corporation	XOM-US
29	J.P. Morgan & Company	JPM.XX9-US				

Source: Data from FactSet.

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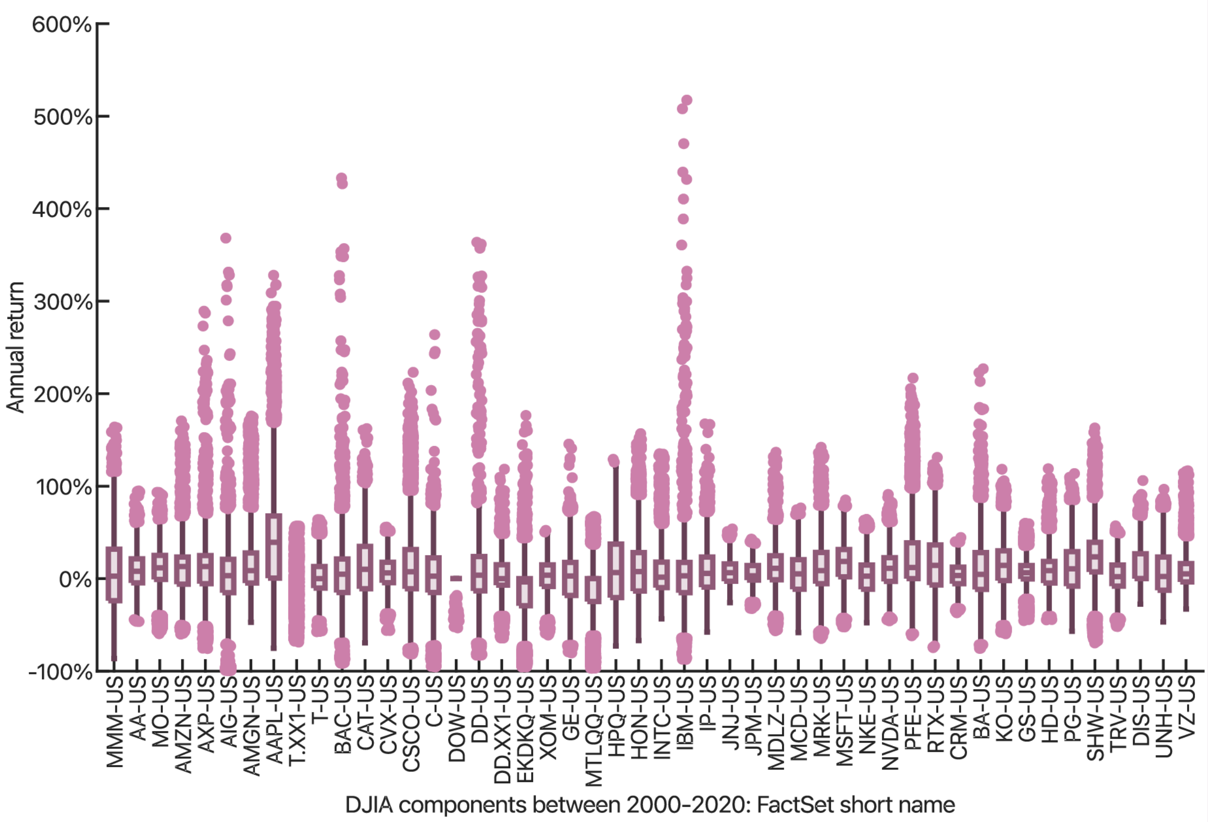
Figure 1

Rolling window annual return for DJIA components: 2000-2020



Source: own elaboration and data from FactSet.

Note: On each day between 2000 and 2020, or 5,134 observations, the trailing annual return for each component belonging to the DJIA at that date is calculated with a rolling window. Annual returns are calculated using the previous 252 trading days ending at the observation date. These trailing returns are descriptive and do not use future prices. See the appendix for the company name.

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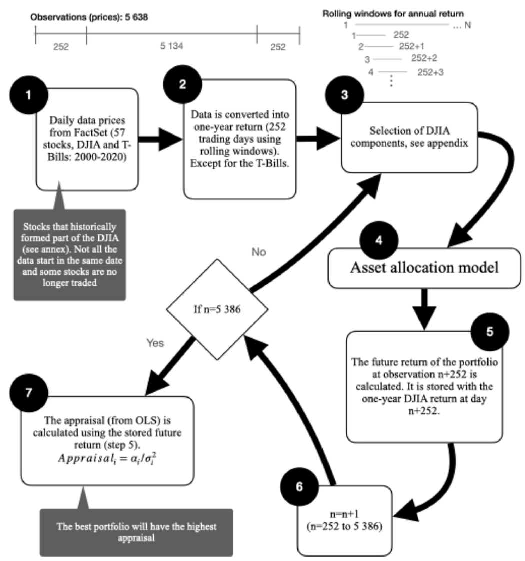
Figure 2

Empirical research process



Source: own elaboration.

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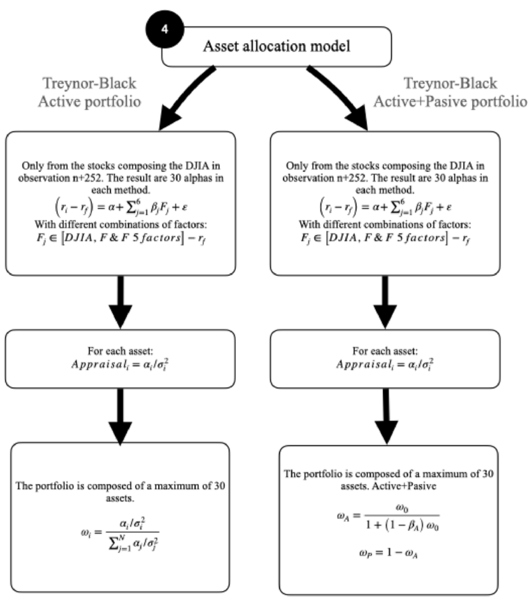
Figure 3

Asset allocation model.



Source: own elaboration.

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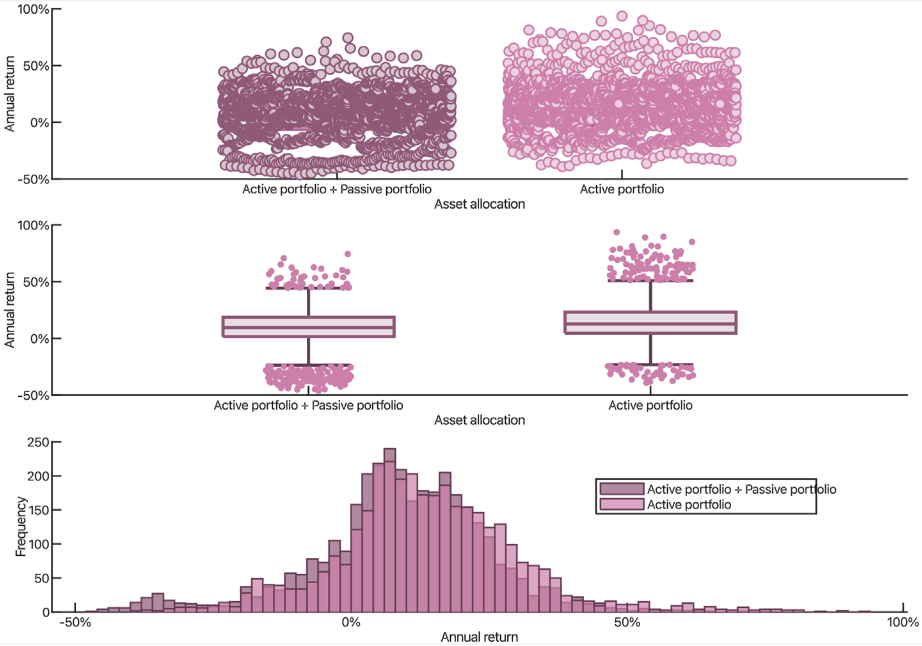
Figure 4

Asset allocation performance: 2000-2020



Source: own elaboration and data from FactSet. Graph one using Campbell (2021).

Note: The Treynor-Black portfolio is composed of two portfolios: the active + passive portfolio. It is observed that the active portfolio has a better performance compared to the passive one. And it does not represent a higher negative volatility compared to the passive one. It improves both negative and positive returns. Table 3 and 4 analyzes the performance of these two types of portfolios.

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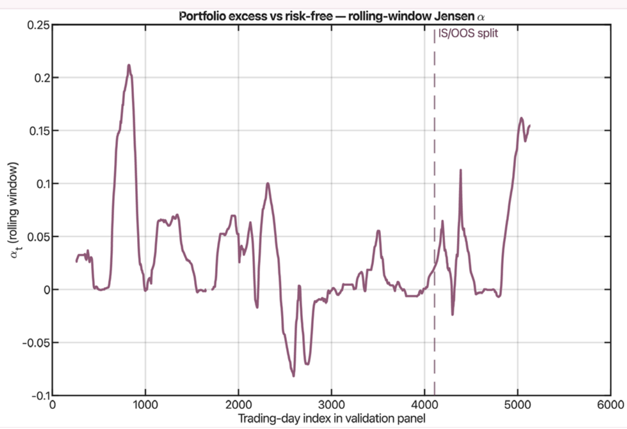
Figure 5

Progressive Jensen alpha - rolling CAPM



Source: own elaboration and data from FactSet using Script_arbitaje.m.

Note: The dashed red line marks the chronological split between the 80% in-sample block and the 20% out-of-sample block. The plotted series is the rolling-window Jensen alpha of the portfolio excess return relative to the DJIA excess return, using 252 trading days at each date.

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