Portfolio Construction Based on Implied Correlation Information and Value at Risk

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- **Abstract:** Value at Risk (VaR) is a commonly used downside-risk measure giving the worst-case asset loss over a target horizon for a given confidence level. Implied correlation from VaR is an alternative form of the correlation coefficient calculated not only based on historic performance, but taking into account a forecast of the worst-loss. Given its importance, here we present a treatment that is accessible to undergraduate students in economics, finance and similar areas with the aim of familiarising the reader with this risk measure. With the use of three case studies we analyse the effect that implied correlation from VaR has on portfolios of increasing asset size. The VaR of each asset is calculated as well as a mean implied correlation, $\bar{\rho}$, which is used to adjust the original portfolio's invested fractions in order to view the shift in risk and return. We track comparative portfolios over a 50-day period to identify trends between portfolio type and risk encountered.

- **Keywords:** Implied correlation, Value at Risk, VaR, Portfolio construction, Risk.
- **JEL Classification:** G11, C60, D81.

- **Resumen:** Valor en Riesgo (VaR) es una medida usada comúnmente para establecer, dado un nivel de confianza, el peor caso de pérdidas en activos. La correlación implícita obtenida a partir de VaR es una forma alternativa del coeficiente de correlación calculada basándose en rendimiento histórico y en un pronóstico de la peor pérdida. En este trabajo presentamos un tratamiento accesible para estudiantes de economía, finanzas y áreas afines con el objetivo de familiarizar al lector con este estimador de riesgo. Con el uso de tres estudios de caso analizamos el efecto que la correlación implícita apartir de VaR tiene en carteras de tamaño creciente. Calculamos el VaR de cada activo así como la media de correlación

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implícita. Dicho valor es usado para ajustar las fracciones del presupuesto en la cartera original. Hacemos un seguimiento comparativo de carteras en un plazo de 50 días para identificar tendencias entre el tipo de cartera y riesgo encontrado.

- **Palabras clave:** Correlación implícita, Valor en riesgo, Construcción de portafolios, Riesgo.
- **Clasificación JEL:** G11, C60, D81.
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Introduction

Value at Risk, or VaR, is a commonly used downside-risk measure giving the worst-case asset-loss over a target horizon for a given confidence level (Jorion, 2001). VaR came into prominence in 1993 when it was unveiled as part of JP Morgan's revolutionary RiskMetrics system (Dowd, 2002) which was designed to give a daily one page summary of all risk across the bank's trading portfolio, with VaR being one of its key elements. Today, VaR is a mainstay of financial risk management; its relative simplicity ensures its popularity in executive environments. Given its importance, we believe that presenting an accessible discussion of the implementation of a portfolio construction methodology will be of great benefit to both newcomers to the area as well as seasoned investors.

When investors face the question of the assets they are interested in holding in efficient portfolios, one of the most important inputs they take into account is the correlation between the assets (Elton et al., 2008; Strong, 2008). It is usually the case that correlation is treated as a constant and unconditional variable, however it is easy to see that it actually varies through time and several studies have provided evidence to that effect (Von Furstenberg and Jeon, 1989; Erb et al., 1994; Longin and Solnik, 1995). In this paper, we use the concept of *implied correlation* to provide a more accurate measure of VaR and present some specific examples of its applicability. The idea is to use implied correlation as a method to estimate the future correlation as a way to improve on the direct use of historical information considering the correlation as a constant. VaR based on implied correlation is an estimator that is not widely discussed and the main objective of this paper is to provide some working examples that enable interested readers to understand this measure, use it and improve on it. With that in mind, the topics addressed here are presented following considerations that can be found in introductory literature to the subject so as to make the material relevant and suitable to a wider readership. We have decided to use some straightforward case studies to demonstrate the implementation of VaR based on implied correlation and as such the results are applicable to those case studies. Nonetheless, we are convinced that a better understanding of this measure by a wider audience is of benefit to all.

▪ *Theoretical Framework*

Let us start with a discussion about VaR itself. As mentioned above, VaR can be described as the maximum potential asset loss, over a given time period for a particular confidence level. VaR can be identified through a quantile of a return distribution function and therefore the VaR of a portfolio is simply a percentile of its return distribution over a fixed time horizon, Δt (Holton, 2003; Jorion, 2001). Value at Risk is therefore equal to the smallest number k in the real numbers $\mathbf{R}$ such that the probability of loss L greater than k over Δt, $P(L>k)$, is at most equal to $(1-\alpha)$, where α is the confidence level; in mathematical terms:

(1) $$VaR=\{min\ k\in R\,|\,P(L>k)\ \le(1-\alpha)\}$$

The information that VaR provides to investors is generally interpreted as a measure of the risk they face (Christoffersen et al., 2001). In other words, for an investor or an institution to require, for instance, a time horizon of 10 days and a confidence level of 99%, we end up with a critical value for VaR of 0.01 of the probability distribution of changes in the market (Duffie and Pan, 1997).

Together with VaR, the idea of correlation has become ubiquitous in the area of finance and the latter is typically used as a measure of dependence between different financial instruments and one that can be used in the problem of portfolio selection and construction (Embrechts, 2002; Campbell et al., 2001). In general terms the correlation between two assets provides us with a measure of how well these two assets move in conjunction with one another and its interpretation is quite straightforward given the fact that its values are bounded between -1 and 1 (Brigham and Ehrhardt, 2013). The correlation coefficient, ρ_{ij}, between assets i and j is expressed as a ratio between the covariance, σ_{ij}, of the two assets under consideration and the product of their standard deviations. In the case of two assets, we have that the correlation coefficient is given by:

(2) $$\rho_{12}=\frac{\sigma_{12}}{\sigma_1\sigma_2}$$

and the covariance is defined as usual:

(3) $$\sigma_{12}=\frac{1}{N}\sum_{i=1}^{N}(R_{1i}-\overline{R}_1)(R_{2i}-\overline{R}_2)$$

where $\overline{R}_i$ represents the average (expected) return of asset i, and standard deviation is the square root of the asset variance.

In general, the methods used for the calculation of correlation rely in greater part on the calculation of the Spearman correlation coefficient based on historical data (Rees, 2001). In the case of correlation forecasting, the most common methodology is the use of an exponentially weighted moving average correlation estimator (Hull and White, 1998) taking into consideration the variation of correlation with time. It is important to mention that in the cases mentioned above the forecast is based still in historical values. In the next section we will show how to use the combination of the assets in a portfolio to measure a correlation implied by the assets themselves in combination.

▪ *Portfolio VaR and Implied Correlation*

In order to motivate the discussion of the use of implied correlation in Value-at-Risk calculations let us consider the simplest of portfolios that can be constructed, i.e. a two-asset portfolio with weights x_1 and x_2 and complying with the full investment constraint (Ning, 2007) such that $x_1+x_2=1$. The distribution of price changes of each of the assets in this portfolio, also known as risk factors (Cotter and Longin, 2007) can be estimated to evaluate the individual Value-at-Risk of the assets. We can denote these risk factors as VaR_1 and VaR_2. The Value-at-Risk of the portfolio, VaR_p, can then be computed using an aggregation formula as done by Cotter and Longin (2007), using Markowitz's portfolio theory (Markowitz, 1952), and in particular the formulation of risk. The authors described the portfolio VaR as a weighting of each asset's worst-case loss, as opposed to variance in the case of risk:

$$(4) \qquad VaR_p = (x_1^2 VaR_1^2 + x_2^2 VaR_2^2 + 2x_1 x_2 \rho_{12} VaR_1 VaR_2)^{1/2}$$

where x_i is the invested fraction in asset i and VaR_i is the Value at Risk of asset i. We would like to point out the similarity between expression (4) above and the risk for a two-asset portfolio (Markowitz, 1952). Please note that equation (4) uses explicitly the correlation coefficient of the asset price changes.

In this work, we are interested in extending the portfolio from two to N assets, and thus the following portfolio VaR formula is obtained:

$$(5) \qquad VaR_p = \left(\sum_{i=1}^{N} x_i^2 VaR_i^2 + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} x_i x_j \rho_{ij} VaR_i VaR_j \right)^{1/2}$$

By noting the correlation term, ρ_{ij}, appearing in the portfolio VaR formula, the implied correlation may be defined. In this case, instead of measuring the relationship between two assets based on past performance, by rearranging the expression for the portfolio VaR, the implied correlation takes into account a forecast of worst-case loss as well. Hence, it is more a measure of the implied correlation between assets within the portfolio in light of the worst-case scenario and it can be used as a measure of future (short-term) correlation. To simplify calculations, a mean correlation value ($\bar{\rho}$) can be used in the equation (5) (Aneja, 1989), replacing individual pairwise terms. This enables the common correlation term to be directly obtained. Hence, the mean implied correlation is obtained in an N -asset case as:

(6)
$$\bar{\rho} = \frac{VaR_p^2 - \sum_{i=1}^{N} x_i^2 VaR_i^2}{2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i x_j VaR_i VaR_j}$$

The difficulties involved in estimating the required inputs to calculate correlation coefficients are well-known and the number of required estimates make it rather impractical. Attempts at addressing these difficulties have been put forward, including the Single Index Model (SIM) (Sharpe, 1963) for instance. Aneja et al. (1989) show a procedure to estimate an average correlation coefficient, which in turn produces better forecasts of the future correlation matrix than those obtained from SIM or the full historical correlation matrix. We note that the expression in Equation (6) is valid within the framework of Markowitz theory (Markowitz, 1952) and under normality conditions. In this work, an implied correlation value will be investigated and applied to portfolios of increasing asset size with the aim of assessing whether its consideration can offer advantages in terms of portfolio risk perception. We would like to note that our aim is to provide the interested reader with formulations that can further be refined and exploited, rather than a definitive new tool. Taking that into account, we believe that presenting a brief summary of VaR computation formalisms is of advantage, particularly for newcomers to the area. Invested fractions will be calculated using a traditional minimum risk approach, with implied correlation from VaR then used to influence the invested fractions and thus highlight any shifts in risk expectations.

■ *Computing VaR*

As we have mentioned before, given how ubiquitous VaR has become in the financial industry, we would like to provide a succinct account of some methods to calculate this measure. We are convinced that this will be of benefit to students in Economics, Finance and Business Administration as well as practitioners outside these areas but who are interested in the topic. In general, there are different approaches to the calculation of VaR and here we outline some of them.

Delta-Normal

The Delta-Normal method is underpinned by the key assumption that the returns of the assets being analysed have a normal distribution. This implies that the return of any portfolio made up of these assets can also be described normally (Linsmeier and Pearson, 2000). Using this assumption, the Delta-Normal method is widely regarded as the most simplistic way of computing VaR, especially in cases where historical asset data is readily available. The approach is particularly appropriate for computing the VaR of portfolios made up solely of assets known to have normally distributed returns. The Delta-Normal VaR can then be stated as:

$$VaR_{\alpha}^{\%} = -(C - Z_{\alpha}\sigma_{p}) \tag{7}$$

where α is the confidence level associated with the VaR, Z is the critical value of the specified confidence level according to the standard normal distribution, σ_p is the standard deviation of portfolio returns expressed as a percentage and C is the expected portfolio return (change in value) over the VaR time horizon. This is frequently assumed to be zero in cases where the time horizon is small, e.g. daily.

The critical value Z of the confidence level is used to scale the standard deviation of returns, hence finding the point at which a given percentage of the returns should lie after. It must also be highlighted that Equation (7) gives a percentage value of VaR, and can therefore be multiplied by the latest portfolio value, S, in order to obtain a monetary value of VaR:

$$VaR_{\alpha} = S(Z_{\alpha}\sigma_{p}) \tag{8}$$

Historical Simulation

The Historical Simulation method relies solely on the past as an indicator of what may occur in the future. It uses the idea that stock returns are random in nature and therefore past returns are a fair representation of the future. The key in the Historical Simulation approach is that each value in the past is given equal weight (Beder, 1995). No other assumptions or inputs are required to compute VaR; the method does not rely on valuation models and is not subjected to the risk of the models being wrong (Jorion, 2001). This ensures the Historical Simulation approach to be relatively simple to implement and understand. The procedure can be described as follows:

1. Calculate periodic returns (as percentages) for all of the assets held in the portfolio. The return period should be the same as the time horizon for which VaR is being calculated.
2. Calculate periodic portfolio returns by summing the multiplication of each asset and its associated invested fraction.
3. Order the set of portfolio returns from most negative to most positive.

4. Calculate how far into the set of ordered returns the VaR will occur (as a percentage of the total data points) by subtracting the decided confidence level percentage from one. Call this x%.
5. Apply x% to the total amount of data points, calculating data point d.
6. Observe the value occurring at d (in the ordered data set). This is the Historical Simulation VaR with a confidence level of (1-x%).

Effectively the value reached should be a portfolio loss which, over the historical period being analysed, will have only been exceeded (negatively) x% of the time.

Monte Carlo Simulation

Monte Carlo Simulation is a commonly used technique that can be applied in many different ways to help model a system, whether physical or mathematical (Binder, 2006). In short, it can be defined as a stochastic problem-solving tool that uses random variables to approximate the probability of certain outcomes by running multiple trial runs.

Monte Carlo procedures are often carried out in cases where data relating to a problem is not available in its entirety. Monte Carlo simulation will then be used to generate random data, representative of the known properties, which imitates the system in question (Binder, 2006; Krauth, 2006). This creates a large set of data from which results can be obtained. Trends are more likely to be confidently identified where ample data is present.

Monte Carlo Simulation VaR uses the known or approximate properties of asset return data in order to generate random data sets from which the VaR can be observed. In terms of the way the VaR is measured, the Monte Carlo Simulation differs very little from the Historical Simulation except the asset and portfolio returns used are created by random draws from the pre-specified stochastic process as opposed to being calculated from the historical prices (Jorion, 2001).

It is usually the case that the Monte Carlo simulation method for calculating VaR relies on the estimation on correlation between assets captured in the correlation between deviates. This in turn means that there is a need to generate correlated random variables (Jorion, 2001) obtained with the aid of Cholesky decomposition (Miranda and Fackler, 2002). This methodology poses a recurrence issue, as a correlation is needed to generate the implied correlation. Instead, in this work we use Principal Component Analysis (PCA) to calculate VaR (Brummelhuis et al., 2002). PCA enables the construction of random variables to be uncorrelated with one another, while describing a large proportion of the variability of the asset prices in question. The principal components are orthogonal which means that they are uncorrelated and so their covariance matrix is simply the diagonal matrix of their variances and thus they can be simulated independently without the need for Cholesky decomposition and with the added advantage of cutting down the computational time needed.

▪ *Case Studies*

Methodology

In this work, we have carried out three case studies, in which portfolios of increasing asset size are formed. The main aim in each case is to calculate the implied correlation using VaR, and then use this to recalculate minimum risk portfolios. Having assessed each of the VaR methods mentioned in the previous section, we have used the Historical Simulation and Monte Carlo Simulation approaches for the analysis carried out in this paper in order to inform the calculation of implied correlations. Delta-Normal, although simple to implement, relies too much on the asset return distribution being normal. Therefore in cases where this may not be the case, it could lead to inaccuracies in the VaR calculated. The case studies are based on stock data over a period of five years from 30th October 2006 to 28th October 2011. Price data was collected using DataStream (Datastream International, 2012). We calculated returns to form the basis of this analysis. In the first instance, we used used five years of returns calculated to construct a historical variance-covariance matrix that allows us to come up with an initial minimum variance portfolio based solely on return information. The main aim of the work is to calculate, using VaR, implied correlation and then use this to recalculate the minimum risk portfolios obtained as described above. The recalculated weights are used to assess the risk and return characteristics of the new portfolios taking into account the VaR information via the implied correlation given by Equation (6).

More specifically, for each case study, we construct the appropriate minimum risk portfolio, comprised of the company stocks considered. We then calculate the daily VaR (with data over 5 years), with a 95% confidence level, of each stock in the portfolio individually and the portfolio itself, using first the Historical Simulation method and then the Monte Carlo Simulation method. Using these VaR values and the minimum risk portfolio invested fractions the mean implied correlation, $\bar{\rho}$, is computed. Using the relationship between covariance, standard deviation and correlation (Equation (2)), revised covariance values can be calculated using the implied correlation. A new variance-covariance matrix is then defined and used in recalculating a minimum risk portfolio. As two methods of calculating VaR are used, this means two different values of implied correlation are calculated and in turn two revised minimum risk portfolios are produced in each case to compare to the original. Conclusions are then drawn on the results found from this approach. We consider 3 case studies, calculating 9 portfolios: 3 minimum variance portfolios, 3 implied correlation portfolios using Historical Simulation VaR and 3 implied correlation portfolios using Monte Carlo VaR. Table 1 shows a summary of the assets included in each case study. Each of the case studies is discussed in Sections 4.2, 4.3 and 4.4 respectively. It should be noted that all assets being used for the investigations are the stock of companies competing in a variety of market sectors.

Finally, we are also interested in assessing the effect that the use of implied correlation from VaR will have in the rebalancing of the portfolio over time. In order to do that, we start with the portfolios calculated using the five-year data described above and carry on a proforma track record of each of the portfolios, with a rebalancing period of 10 days (comulatively) over a period of four months, using therefore data from 31st October 2011 to 13th January 2012. This analysis is presented in Section 4.5.

Case Study 1

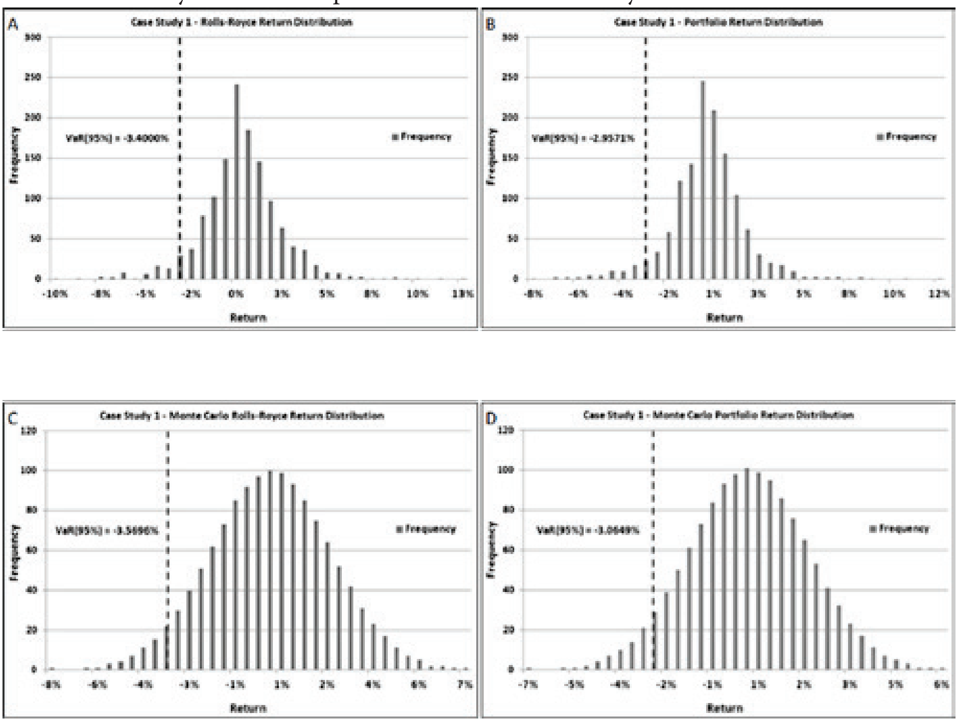
This case study is a two-asset portfolio based on Rolls-Royce and General Electric. The minimum risk portfolio weightings for the starting portfolio are: Rolls-Royce 58.10%, General Electric 41.90%. These invested fractions, over the five-year period from 2006 to 2011 as stated above, correspond to a portfolio risk of 1.8794% and a return of 0.0355%. With this information we obtain the VaR for the assets and the portfolio. The VaR value for Rolls-Royce stock over the five-year period turned out to be -3.40% using the Historical Simulation approach as can be seen from Figure 1.A. In turn, the VaR using the Monte Carlo Simulation was approximately -3.5696% as shown in Figure 1.C. Using similar calculations for General Electric stock, we are able to calculate its VaR: -3.8069% for the Historical Simulation and -4.0973% for the Monte Carlo Simulation.

Table 1
Number of assets and companies used in the case studies carried out for this work

Case Study	Number of Assets	Assets
1	2	Rolls-Royce, General Electric
2	3	As in case study 1 + Barclays
3	10	As in case study 2 + BP, British American Tobacco BSKYB, Centrica, GlaxoSmithKline, Tesco, Vodafone

The assets used in each case study are cumulative from one case study to the next.
Source: Own elaboration.

Figure 1
Case Study 1: Two-asset portfolio based on Rolls-Royce and General Electric



Panels A and C show the VaR obtained from the Historical Simulation approach over a five year period for Rolls-Royce and General Electric, respectively. Panels B and D show the VaR calculated from Monte Carlo Simulations over a five-year period for Rolls-Royce and General Electric, respectively.
Source: Own elaboration.

With the values mentioned above we can now find values for the implied correlation that will allow us to calculate new variance-covariance matrices to produce revised minimum variance portfolios. The rebalanced portfolios had very similar compositions:

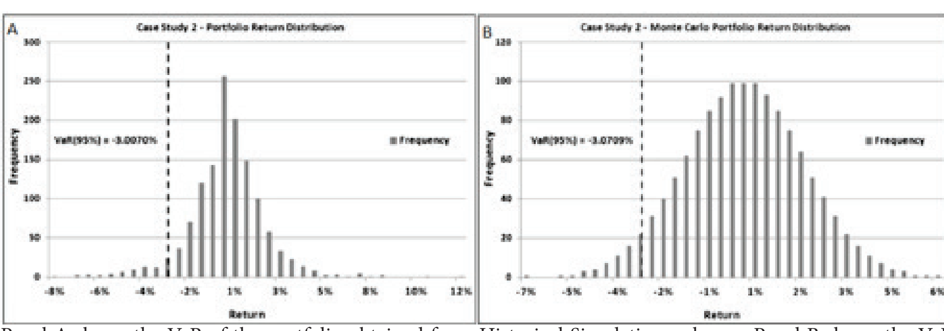
- For the Historical Simulation: Rolls-Royce 58.85% and General Electric 41.15%
- For the Monte Carlo Simulation: Roll-Royce 58.06% and General Electric 41.94%

These weights and the implied VaR calculation give us the following results: for the Historical Simulation we obtain a portfolio VaR of -2.8771%, whereas the Monte Carlo Simulation turns out to be -3.0649%. These results can be seen in Figures 1.B and 1.D, for the Historical Simulation and Monte Carlo Simulation, respectively. The new portfolio generated based on implied correlation from Monte Carlo Simulation VaR perceives the risk to be lowest, at 1.8771%, against comparable portfolios considered here, i.e. Minimum Risk and implied correlation based on Historical Simulation VaR. This alone shows that by considering implied correlation, an alternative portfolio can be generated which may offer advantages in terms of risk perception. The invested fractions of the portfolio formulated using implied correlation from Historical Simulation VaR correspond to the highest risk level out of the portfolios calculated. As expected though, this higher risk level does offer the greatest return of 0.0361%. The returns offered by the implied correlation Monte Carlo Simulation VaR portfolio and the minimum risk portfolio are very similar (at around 0.035%). This shows that in comparison (of just risk/return values) the Monte Carlo portfolio offers better value to the investor (in terms of risk perception), offering a slight reduction in risk for the same level of return. In reality, the three methods used to generate portfolios are alternative ways of getting an answer to the optimisation problem. An investor will never have to choose between such portfolios, although the variation in risk and return values may play a key part in choice between alternative investments (a portfolio may be recalculated, using implied correlation, with the aim of showing a lower risk than a different portfolio).

Case Study 2

Case Study 2 is a three-asset portfolio comprised by Rolls-Royce and General Electric (as in case study 1) together with Barclays. The minimum risk portfolio weightings for the starting portfolio are as follows: Rolls-Royce 59.21%, General Electric 42.35% and Barclays –1.56%. The small, short sold invested fraction attributed to Barclays demonstrates the volatility of the stock over the period of returns used (the standard deviation is significantly higher than that of Rolls-Royce and General Electric) - it appears to be a risky asset, and the weighting reflects this. The risk associated with these initial portfolio weightings, measured over the five-year historical return period is 1.8782%. The portfolio return for the same period was calculated to be 0.0362%.

Figure 2
Case Study 2: Three-asset portfolio comprised by Rolls-Royce, General Electric and Barclays



Panel A shows the VaR of the portfolio obtained from Historical Simulations whereas Panel B shows the VaR obtained from the Monte Carlo approach.

Source: Own elaboration.

The revised portfolio invested fractions calculated from the Historical Simulation VaR implied correlation were as follows: Rolls-Royce 61.09%, General Electric 41.92% and Barclays with a further short-selling at -3.01%. As found with Case Study 1 the historical simulation revised portfolio is perceived riskier, at 1.9487%, than the minimum risk portfolio, and it offers a greater return of 0.0375%. The same revised portfolio, but this time using Monte Carlo Simulation VaR implied correlation had weights of: Rolls-Royce 56.92%, General Electric 40.95% and Barclays 2.13% - Notice that in this case the short-selling for Barclays has gone away. This portfolio had associated five-year risk of 1.8751% and a return of 0.0348%.

The results of the Historical Simulation and Monte Carlo Simulation approaches for the VaR of this portfolio can be seen in Figures 2.A and 2.B, respectively. The Monte Carlo Simulation VaR implied correlation portfolio, as in Case Study 1, perceives the lowest risk compared with the other two portfolios. This time though the lower risk achieved is at the cost of the associated return, which is also lowest in comparison to the other two portfolios. Further highlighting the natural relationship between risk and return, the Historical Simulation VaR implied correlation portfolio carried the highest risk as well as return.

Case Study 3

Case Study 3 corresponds to a 10-asset portfolio comprised by Rolls-Royce, General Electric, Barclays, BP, British American Tobacco, BSkyB, Centrica, GlaxoSmithKline, Tesco and Vodafone. We have calculated the VaR for the stocks mentioned above and which are summarised in Table 2.

Table 2
Daily VaR (95% confidence level) from five years of data from October 2006 for the companies included in Case Study 3

Stock	Historical Simulation VaR	Monte Carlo Simulation VaR
Rolls-Royce	-3.4000%	-3.5696%
General Electric	-3.8069%	-4.0973%
Barclays	-5.9957%	-7.4426%
BP	-2.9540%	-3.2804%
British American Tobacco	-2.3084%	-2.5714%
BSKYB	-2.8388%	-3.0213
Centrica	-2.5767%	-2.8530%
GlaxoSmithKline	-2.2286%	-2.4066%
Tesco	-2.5309%	-2.7237%
Vodafone	-2.8727%	-3.0603%

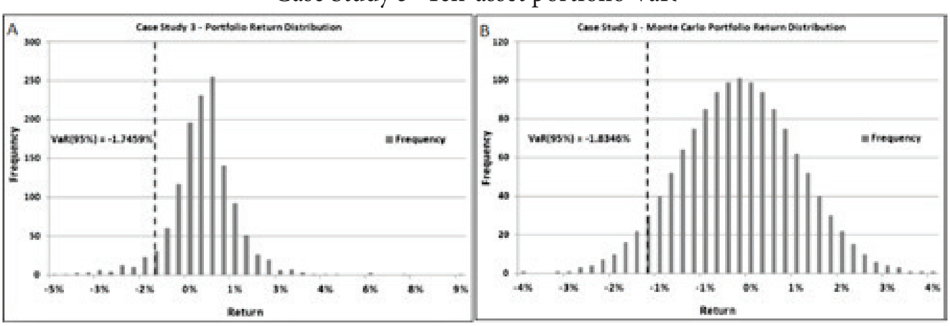
Source: Own elaboration.

Table 3
Weights of the three different portfolios considered in Case Study 3

Stock	Minimum Risk	Implied Correlation Historical Simulation VaR	Implied Correlation Monte Carlo Simulation VaR
Rolls-Royce	-3.04%	1.31%	1.90%
General Electric	11.50%	-2.61%	-1.78%
Barclays	-5.61%	-9.23%	-8.14%
BP	9.01%	5.95%	6.25%
British American Tobacco	15.33%	21.55%	20.81%
BSKYB	13.54%	9.92%	9.96%
Centrica	13.61%	14.50%	14.23%
GlaxoSmithKline	26.81%	31.03%	29.63%
Tesco	15.01%	18.40%	17.87%
Vodafone	3.84%	9.18%	9.27%

Source: Own elaboration.

Figure 3
Case Study 3- Ten-asset portfolio VaR



Panel A shows the VaR of the portfolio obtained from Historical Simulations. Panel B shows the VaR obtained from the Monte Carlo Simulations.
Source: Own elaboration.

The weights of the different portfolios considered in this case study are shown in Table 3. From the composition of the portfolios it is easy to see how some of the stocks change their weights in the portfolio once the VaR information is incorporated in the calculation. A case in point is that of General Electric: in the minimum variance portfolio this stock has a weight of 11.50%, whereas once the VaR is included in the optimisation of the portfolio, this stock is short sold in both portfolios that consider the implied correlation measure. The opposite behaviour is seen for Rolls-Royce.

The Monte Carlo Simulation VaR implied correlation portfolio once again perceived the lowest level of risk at 1.0777% compared to 1.0963% for the Historical Simulation one. The return associated very close in both cases at 0.031%. The Monte Carlo approach, giving the lowest risk, has provided a consistent result throughout all three case studies. Nonetheless, this can indeed be influenced by the periods analysed, and as we know in the area, future performance must not be inferred from past results.

This case study behaved largely as expected, showing a significant reduction in both risk and return. The reduction in risk was also reflected in the values of VaR - the worst-case portfolio loss was shown to be a lot lower than both the two and three asset cases. Figures 3.A and 3.B show the portfolio VaR for case study 3 using the Historical Simulation (-1.7459%) and Monte Carlo Simulation (-1.8346%) respectively. With these results in mind, considering implied correlation seems to be an adequate alternative way of calculating and even checking a portfolio. The two methods of calculating the measure are showing different extremes of results when applied to a portfolio and compared back to the minimum risk case (Monte Carlo Simulation tends to give lower risk compared to Historical Simulation).

Further Analysis – Rebalancing of the Portfolios

Having distinguished that the use of implied correlation offers an alternative way of calculating a minimum risk portfolio, which may or may not have a lower perceived level or risk, further analysis can be carried out to see which set of portfolio weights actually gives the lowest risk looking back over a time period. This will be tested using the 9 sets of portfolio weightings already calculated over Case Studies 1 to 3 mentioned above plus an additional 36 portfolios based on an expanding time series.

In order to do this, we will take the view that after the initial portfolio weightings are calculated (based on the 5 year return data), they are all then invested in immediately for a ten-day period. After 10 days the risk and return of each portfolio is then calculated. This will give us 9 risk/return values, one each for the minimum risk portfolio, the revised portfolio based on Historical Simulation VaR and the revised portfolio based on Monte Carlo Simulation VaR per case study. A further total of 9 portfolios will then be generated, based on the 5-year plus 10-day data. This time the 20-day risk/return values are calculated for each. We would like to note that this 20-day period of data is inclusive of the 10-day period used earlier with the additional 10 subsequent trading days. We then go on to calculate another set of 9 portfolios, using 5 year plus 20-day data and then calculate the 30-day risk/return points for each of these. This pattern will continue until 5 year plus 40-day portfolios are computed and then applied over a 50-day period, generating risk/return values.

At the end of these calculations there are 45 risk/return values which effectively track the three types of portfolio over a 50-day period (from October 31st 2011 to January 13th 2012). The following figures show the resulting risk/return values plotted per case study: Figure 4 shows all two-asset portfolio points plotted on the same risk-return graph, likewise the 3 and 10 asset portfolio points are shown in Figures 5 and 6, respectively. Each cluster of points plotted is comparative and the further left a point lies, the lower that portfolio's risk is. It should be noted that one cluster of points (containing the minimum risk point, the Historical Simulation VaR point and the Monte Carlo Simulation VaR point) should not be compared to one another with the view of making an investment choice, as each cluster has been measured over a different return period.

The plot in Figure 4, contains the 15 (5 per method) risk/return values calculated for the two-asset portfolio of Case Study 1. As can be seen from each cluster of points, all 3 methods give extremely close values for risk and return, with the points almost indistinguishable from each other. In other words, an investor, with a risk-averse attitude, should be fairly indifferent (in a two-asset case) if given a choice as to how they construct their portfolio, as it is not obvious that either method achieves a lower risk. The cluster of points that lie below the x -axis show a negative return - this is due to the 20-day period over which these were measured. This period was a particularly poor one in terms of the individual asset returns (contributing to the portfolio) with a Rolls-Royce expected (average) return of -0.01% and General Electric expected return of -0.40%.

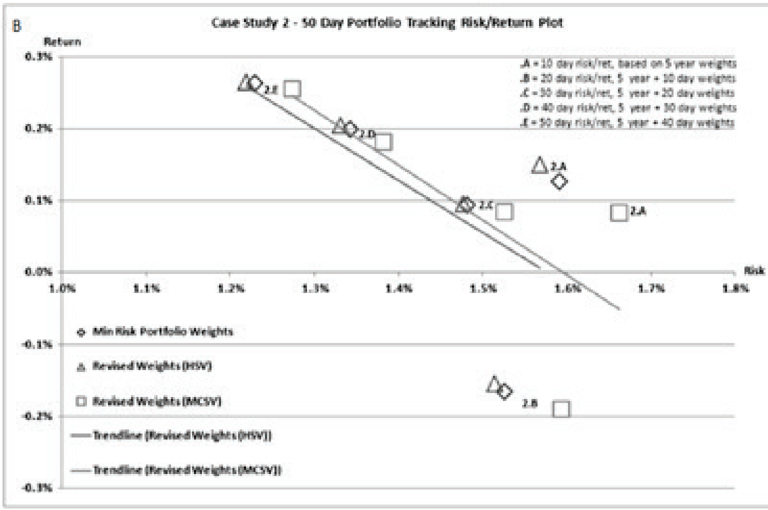
Figure 4
Risk-return values for a 50 day tracking of the two-asset portfolio in Case Study 1

Source: Own elaboration.

Figure 5 is the plot of the risk/return values for Case Study 2, in which the three-asset portfolio was analysed. The first thing to notice is that, despite a similar plotting scale being used, the points are now a lot more dispersed than in the previous case study. The addition of a further asset to the portfolio has had the affect of achieving a much more diverse set of comparative points. These clusters (containing the minimum risk point, the Historical Simulation VaR point and the Monte Carlo Simulation VaR point) are now less obvious to identify compared with the two-asset case (Figure 4). It has therefore been easier to recognise a pattern within the data, in that the portfolios generated using implied correlation from Historical Simulation VaR appear to be consistently achieving the lowest risk for a given level of return over each different measuring period (and often the highest return also). We would like to note that the lines shown in Figure 5 are used merely to guide the eye and should not be though of as best fit or trend lines necessarily. The Historical Simulation line lies furthest to the left and it is therefore understood that this method reduces risk in this three-asset case. The guiding line for the minimum risk portfolio is not shown on this graph as it is not clearly distinguishable from the Historical Simulation line. Although this shows that the points consistently lie extremely close together, it is clear by inspection that the Historical Simulation points do lie further left in each cluster. The Monte Carlo Simulation guiding line demonstrates not only that this method always gave the highest risk level; in each case it also achieved the worst level of return. This method of generating portfolios will have therefore given an investor poor value in comparison to the other two methods, in this particular case. Nonetheless, it can certainly be argued that the results may be of interest for very conservative investors.

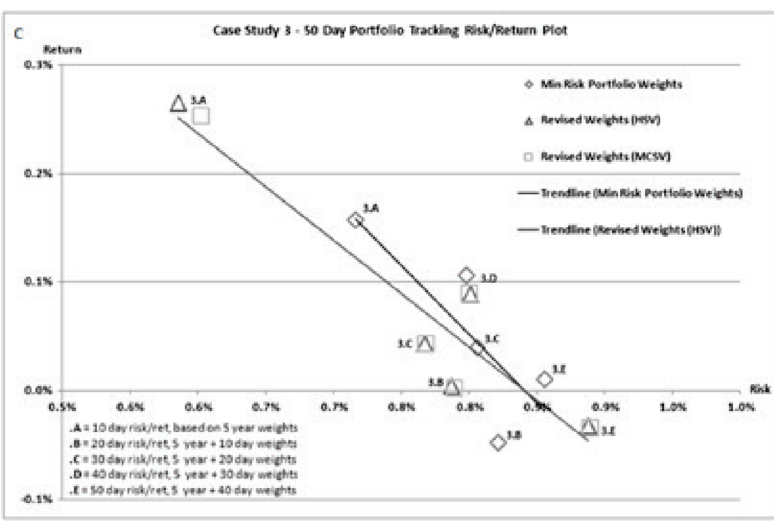
The plot corresponding to the portfolios of Case Study 3 is shown in Figure 6. It shows an even greater dispersion of cluster points. It appears that the more assets held in the portfolio, the greater the difference in the risk/return points achieved by the three different methods. Comparing the two-asset to the ten-asset case confirms this. A possible reason for this increasing dispersion is the uncertainty introduced into the model by the increasing number of assets. In terms of the method giving the most favourable level of risk (from the viewpoint of a risk averse investor) in this case, the results are a lot more mixed compared with Case Study 2. In 3 of the 5 cases, the Historical Simulation VaR implied correlation portfolios have achieved the lowest risk level while in 2 of the 5 cases the minimum risk portfolio has given the lowest risk. Interestingly, in the 40-day measured period the Historical Simulation portfolio actually gave the highest risk. These results are highlighted by the guiding lines plotted (the Monte Carlo Simulation line has not been plotted as it cannot be distinguished from the Historical Simulation line on the scale used). The crossover that occurs around 0.85% risk has been caused by the pair of minimum risk portfolio points that have a superior risk level in comparison to the relevant Historical Simulation points. Similarly to Case Study 2 (the three-asset case) the Monte Carlo Simulation points have tracked the Historical Simulation points very closely, much like the minimum risk portfolio points in that case (see Figure 5).

Figure 5
Risk-return values for a 50-day tracking of the three-asset portfolio in Case Study 2



Source: Own elaboration.

Figure 6
Risk-return for a 50 day tracking of the ten-asset portfolio in Case Study 3



Source: Own elaboration.

Having completed this analysis, comparing the actual risk/return performance over increasing time periods of the minimum risk portfolio to portfolios generated using implied correlation from VaR, we can make the following observations: Although the graph of Case Study 2 (Figure 5) points towards the portfolios generated using implied correlation from Historical Simulation VaR performing the best based on a remit of reducing risk, the two- and ten-asset studies have produced somewhat inconclusive results. This is to be expected given the constraints used to generate these portfolios while keeping the present work within the remit of a publication. It seems a greater amount of data and analysis is needed to study whether or not there are certain conditions that will see one method generating consistently superior portfolio results in terms of the risk level achieved. In this particular case, our aim is to highlight this methodology to the community in the area and expect that these results encourage others to tackle the issues raised here. It should be noted that the way in which the results of each of the methods are viewed will be largely dependent on a particular investor's attitude to risk, and that must be something to always bear in mind. In this work, we have taken the view that minimum risk is favourable, however this is not always the assumption made (Fabozzi et al., 2002).

■ *Conclusions*

In this work we have used an alternative measure for the correlation among assets in a portfolio, namely the implied correlation calculated from Value at Risk information (VaR). Various methods to calculate VaR have been assessed: the Delta-Normal method, the Historical Simulation method or the Monte Carlo Simulation method. We have presented a brief account of the calculation of VaR with these three approaches in the expectation that students in the area get a better picture of the methodologies used. Out of these three methods, the Delta-Normal approach was deemed to have the greatest disadvantage due to an over reliance on the assumption of a normal distribution. The Historical Simulation, which aims to use the past as a representation as what may occur in the future, seems to be an appropriate way of calculating VaR. However, in an evolving world where markets are becoming increasingly volatile this notion may lead to underestimations of risk. This is where the Monte Carlo Simulation can be very powerful in modelling different scenarios, although this does come at a significant computational expense.

The application of implied correlation has been shown as an alternative way of generating a portfolio, expanding on the traditional minimum risk approach. Comparing a standard minimum risk portfolio against the same portfolio influenced by implied correlation, it can be seen that the risk perception is altered based on the consideration of future risk. However, in the portfolios analysed here there is no conclusive evidence that suggests implied correlation portfolios offer a consistently higher/lower risk. This by no means undermines the usefulness of the approach as further analysis should be carried out and we expect that a refinement of the method sheds more light into the issues raised in this work.

A further analysis was carried out in order to see if any of the methods performed consistently in terms of actual achieved risk and return over increasing time periods. In particular, the method whose invested fractions frequently gave the lowest risk was sought. Case Study 2 demonstrated that the implied correlation from Historical Simulation VaR portfolios consistently achieved the lowest risk. The three methods for constructing a portfolio presented in this work, namely the minimum risk approach, implied correlation based on Historical Simulation VaR and implied correlation based on Monte Carlo Simulation VaR implied correlation, should all be considered viable techniques. A key factor in determining which is the so-called *best* approach is the particular investor's attitude to risk. In this work, we have taken the view that that minimum risk is desired and therefore the main priority. However, this view is not always shared across global trading markets, and a different perspective on risk could lead to different conclusions regarding the methods.

The main aim of this paper has been to provide a straightforward implementation of the calculation of VaR based on implied correlation as an alternative estimator for downside-risk in order to familiarise the reader with this risk measure. The results of the case studies presented are very particular to the portfolios described and should not be taken as generalisations. We expect that the discussions here enable interested practitioners to make further considerations, studies and implementations that provide a more general framework to the VaR based on implied correlation.

■ *References*

ANEJA YP, CHANDRA R, GUNAY E (1989). A Portfolio Approach to Estimating the Average Correlation Coefficient for the Constant Correlation Model. *The Journal of Finance*, Vol. 44, No.5, pp. 1435–1438.

BEDER TS (1995). VaR: Seductive but dangerous. *Financial Analysts Journal* Vol. 51, No. 5, pp. 12-24.

BINDER K (2006). Monte-Carlo Methods, in Mathematical Tools for Physicists. Wiley, Weinheim, Germany.

BRUMMELHUIS R, CÓRDOBA A, QUINTANILLA M, SECO L (2002). Principal Component Value at Risk. *Mathematical Finance* Vol. 12, No. 1, pp. 23-43

BRIGHAM E, EHRHARDT M (2013). Financial Management: Theory & Practice. Cengage Learning, Canada.

CAMPBELL R, HUISMAN R, KOEDIJK K (2001). Optimal Portfolio Selection in a Value-at-Risk framework. *Journal of Banking & Finance* Vol.25, Is. 9, pp. 1789-1804.

CHRISTOFFERSEN P, HAHN J, INOUE A (2001). Testing and comparing Value-at-Risk measures. *Journal of Empirical Finance*, Vol. 8, pp. 325-342.

COTTER J, LONGIN F (2007). Implied Correlation from VaR. Available at SSRN: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=996080.

DATASTREAM INTERNATIONAL (2012). Thompson Reuters: http://online.thomsonreuters.com/datastream/

DOWD K (2002). An Introduction to Market Risk Measurement. John Wiley and Sons Ltd, Chester, UK.

DUFFIE D, PAN J (1997). An Overview of Value At Risk. *The Journal of Derivatives* Vol. 4, No. 3, pp. 7–49.

ELTON EJ, GRUBER MJ, BROWN SJ, GOETZMANN WN (2008). Modern Portfolio Theory and Investment Analysis, 8th edn. Wiley, US.

EMBRECHTS, P, MCNEIL, A, STRAUMANN, D (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, 176-223. Cambridge University Press, Cambridge, UK.

ERB CB, HARVEY CR, VISKANTA, E. (1994). Forecasting international equity correlations. *Financial Analysts Journal*, Vol. 6, pp. 32-45.

FABOZZI FJ, GUPTA F, MARKOWITZ M (2002). The Legacy of Modern Portfolio Theory. *The Journal of Investing* Vol. 11, No. 3, pp. 7-22.

HOLTON GA (2003). Value-at-Risk: Theory and practice, Academic Press, California, US.

HULL J, WHITE A (1998). Incorporating volatility updating into the historical simulation method for value-at-risk. *Journal of Risk* Vol. 1, No. 1, pp. 5-19

JORION P (2001). Value at Risk, 2nd edn. McGraw-Hill, California, US.

KRAUTH W (2006). Statistical Mechanics: Algorithms and Computations. Oxford University Press, Oxford, UK.

LINSMEIER T, PEARSON N (2000). Value at Risk. *Financial Analysts Journal* Vol. 56, No. 2, pp. 1–21.

LONGIN F, SOLNIK, B. (1995). Is the correlation in international equity returns constant: 1960-1990? *Journal of International Money and Finance*, Vol. 14, pp. 3-26.

MARKOWITZ H (1952). Portfolio Selection. *The Journal of Finance* Vol. 7, No. 1, pp. 77–91.

MIRANDA MJM FACKLER PL (2002). Applied Computational Economics and Finance. MIT Press, US.

NING H (2007). Hierarchical Portfolio Management: Theory and Applications. PhD Thesis Erasmus School of Economics, Erasmus University Rotterdam, Netherlands.

REES DG (2001). Essential Statistics, 4th edn. Chapman & Hall, Florida, US.

SHARPE WF (1963). A Simplified Model for Portfolio Analysis. *Management Science* Vol. 9, pp. 277–293.

STRONG R (2008). Portfolio Construction, Management, and Protection, 5th, edn. Cengage Learing, Canada.

VON FURSTENBERG GM, JEON BM (1989). International stock price movements: links and messages. *Brookings Papers on Economic Activity* Vol. 1, 125-180.