Temporary stabilization: a Fréchet-Weibull extreme value distribution approach

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- **Abstract:** This paper develops, in a small open economy of pure exchange framework, a stochastic model of exchange-rate-based inflation stabilization plan that is expected to be temporary. Agents have expectations of devaluation driven by a mixed diffusion-jump process where the expected size of a possible devaluation is supposed to have an extreme value distribution of the Fréchet-Weibull type. Consumption and wealth equilibrium dynamics are examined when such a stabilization plan is implemented. It is assumed that financial markets are incomplete, that is, there are more risk factors than risky assets. Finally, the effects of exogenous shocks on economic welfare are assessed.

- **Resumen:** Esta investigación desarrolla, en el marco de una economía pequeña y abierta, un modelo estocástico de un plan de estabilización de precios que toma como un ancla nominal al tipo de cambio y en donde se espera que dicho plan sea temporal. Los agentes tienen expectativas de devaluación conducidas por un proceso mixto de difusión con saltos en donde el tamaño esperado de una posible devaluación tiene una distribución de valores extremos del tipo Fréchet-Weibull. Las dinámicas de equilibrio del consumo y la riqueza son examinadas cuando un plan de estabilización es implementado. Se supone que los mercados financieros son incompletos, es decir, hay más factores de riesgo que activos riesgosos. Por último, se evalúan los efectos de choques externos en el bienestar económico.

- **Keywords:** inflation stabilization, extreme values.

- **JEL Classification:** F31, F41.

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■ *Intrdouction*

Between 1900 and 2000, emerging economies managing a fixed exchange-rate, a fixed rate of devaluation, an adjustable band or a convertibility plan, like Brazil, Ecuador, Thailand, Korea, Indonesia, Russia, Bosnia, Turkey, among others, had a major financial crisis. These regimes have simply not proved viable over time, especially for countries integrated or integrating into international capital markets. In most cases, the public in these countries anticipated that the stabilization plan was going to be temporary, resulting in a large expansion of consumption of durable goods and an extreme devaluation. Surprisingly, in 2008, as of April 31, according to the IMF, at least 88 of the world countries have one of the above exchange-rate regimes, as it is shown below in Table 1. The lessons, from these episodes, that should be taken into account by policymakers when devising a corrective devaluation, especially if financial markets are incomplete, is that public expectations and consumption dynamics generated by exchange-rate-based stabilization plans may increase imports producing unsustainable deficit in the current account of the balance of payments.

The experience in most of these countries brings the credibility of stabilization programs to our attention. In most cases, the public was skeptical about government's commitment to defend a regime where the exchange rate was taken as a nominal anchor. In many cases, the final outcome was a consumption boom and an extreme devaluation putting an end to an exchange-rate-based stabilization program.[4]

In Calvo and Reinhart's (2002) paper "Fear of floating", it has emphasized that many countries that claim to have floating exchange rates do not, in practice, allow the rate to float freely, but instead use interest rate and intervention policies to affect its behavior. From this, they draw two conclusions: in first place, that it is incorrect to claim that countries are moving away from adjustable-peg systems. Secondly, that since countries hanker after fixed exchange rates for good reasons, they would be well advised to adopt hard pegs. In their paper, Calvo y Reinhart also investigate whether countries are, indeed, moving as far to the corners as official labels suggest. Since verifying the existence of a hard peg is trivial, their focus is on the other end of the flexibility spectrum. Specifically, they examine whether countries that claim they are floating their currency are, indeed, doing so.

Policymakers now warn against the use of a fixed rate of devaluation or an adjustable band in countries open to capital flows. This belief that intermediate regimes between fixed exchange-rate and free floating are unsustainable is known as the bipolar view (see, for instance, Fischer, 2001). The proportion of IMF members with intermediate arrangements fell during the 1990s, while the use of hard pegs and more flexible arrangements grow. Also, Table 2 shows the evolution of de facto exchange -rate arrangements.

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4 The inflation stabilization programs, which took place in Latin America in the 1990's, have been widely documented. We direct the reader to the references contained in Calvo and Végh (1998). In Latin America, the exception was the case of Peru, that left the nominal exchange rate float. Moreover, stabilization in Argentina was not based on an intermediate regime but in a hard peg; a currency board was used.

Table 1
Countries with an Exchange Rate Anchor
(Source IMF, as April 31, 2008)

(Currency board, 13 countries)			
U.S. dollar	Euro	Composite	Other
(8)	(4)	(0)	(1)
ECCU	Bosnia & Herzegovina		Brunei D.
Antigua & Barbuda	Bulgaria		
Djbouti	Estonia*		
Dominican Republic	Lithuania*		
Grenada			
Hong Kong, SAR			
St. Kitts & Nevis			
St. Lucia			
St. Vincent & Grenadines			

(Conventional peg, 44 countries)			
U.S. dollar	Euro	Composite	Other
(15)	(19)	(5)	(5)
Aruba	Cape Verde	Fiji	Bhutan
Bahamas, The	Comoros	Kuwait**	Lesotho
Bahrain	Denmark*	Libya	Namibia
Barbados	Latvia*	Morocco	Nepal
Belize	Sâo Tomé & Principe	Samoa	Swaziland
Eritrea	WAEMU		
Jordan	Benin		
Maldives	Burkina Fasso		
Netherlands Antilles	Côted’Ivore		
Oman	Guinea-Bissau		
Saudi Arabia	Mali		
Turkmenistan	Niger		
United Arab Emirates	Senegal		
Venezuela	Togo		
	CEMAC		
	Cameroon		
	Central African Rep.		
	Chad		
	Congo, Rep. Of		
	Equatorial Guinea		
	Gabon		

WAEMU: West African Economic and Monetary Union. CEMAC: Monetary and Economic Community of Central Africa. FYR: Former Yugoslav Republic. ECCU: Eastern Caribbean Currency Union

* The member participates in the European Exchange Rate Mechanism (ERM II).

** The exchange rate arrangement was reclassified retroactively, overriding a previously published classification.

(Stabilized arrangement, 16 countries)			
U.S. dollar	Euro	Composite	Other
(12)	(2)	(2)	(0)
Azerbaijan	Croatia	Iran, Islamic Rep. of	
Bangladesh	Macedonia, FYR	Syrian Arab Rep.**	
Bolivia**			
Cambodia			
China			
Guyana			
Honduras			
Iraq**			
Lebanon			
Suriname			
Trinidad and Tobago			
Vietnam**			

(Crawling peg, 3 countries)			
U.S. dollar	Euro	Composite	Other
(2)	(0)	(1)	(0)
Nicaragua		Botswana	
Uzbekistan			

(Craw-like arrangement, 2 countries)			
U.S. dollar	Euro	Composite	Other
(2)	(0)	(0)	(0)
Ethiopia			
Kazakhstan			

(Pegged Exchange rate within horizontal bands, 2 countries)			
U.S. dollar	Euro	Composite	Other
(0)	(0)	(2)	(0)
		Belarus	
		Tonga	

(Other management arrangement, 8 countries)			
U.S. dollar	Euro	Composite	Other
(3)	(0)	(5)	(0)
Angola		Algeria	
Costa Rica		Russian Federation	
Liberia		Singapore**	
		Solomon Islands	
		Vanuatu	

** The exchange rate arrangement was reclassified retroactively, overriding a previously published classification.

Table 2
Evolution of de facto exchange-rate arrangements (%)

Regime	1990	1995	2000	2003	2008
Hard pegs	69.2	58.9	41.4	56.5	66.5
Intermediate pegs	15.1	24.9	33.9	18.3	21.3
Floating	15.7	16.2	24.7	25.1	12.2

Source: IMF

Studies in the literature on temporary stabilization based on a semi fixed exchange-rate that have considered a stochastic setting are, for instance, Drazen and Helpman (1988) examining stabilization with exchange-rate management under uncertainty, Calvo and Drazen (1997) contemplating uncertainty in the permanence of economic reforms,[5] and Mendoza and Uribe (1996) and (1998) modeling exogenous and endogenous probabilities of devaluation, respectively. On the other hand, Venegas-Martínez (2000) and (2001), (2006a) has studied exchange-rate-based stabilization with imperfect credibility; Venegas-Martínez (2006b) has examined the impact of fiscal policy exchange-rate-based stabilization with imperfect credibility, and Venegas-Martínez (2010) has valued the real option of waiting when consumption can be delayed in exchange-rate-based inflation stabilization program. It is also important to highlight the work of Uribe (2002) and Uribe and Mendoza (2000) regarding the explanation of the order of magnitude of the unexpected consumption booms and the incorporation of uncertainty in the analysis of temporary stabilization plans. Also, in Mendoza (2001) the benefits of dollarization when the stabilization policy lacks credibility and financial markets are imperfect are examined. Finally, Akgiray and Booth (1988) have made clear that both monetary policy and targets involve different parameters of the exchange-rate distribution, this in the spirit of the Lucas' critique.

While the above literature has provided considerable theoretical advancement, there are some issues on credibility and uncertainty that still need to be explained, as remarked in: Helpman and Razin (1987), Kiguel and Liviatan (1992), Végh (1992), and Rebelo and Végh (1995). First, in the existing models, it is missing a plausible explanation of the lack of credibility. Secondly, most models forget that what makes a stabilization inflation program temporary is uncertainty.

This paper develops, in small open economy setting, a stochastic model of exchange-rate-based stabilization recognizing the role of extreme movements in the dynamics of the expectations of devaluation. It is assumed that the expectations of devaluation follow a mixed diffusion-jump process where a Brownian motion drives the rate of devaluation and a Poisson determines the number of possible devaluations. The expected size of a possible devaluation is supposed to have an extreme value distribution of the

5 Though Calvo and Drazen (1998) focus on the duration of economic reforms, their results can be translated into an exchange-rate-based disinflation context. It is also important to point out that while Drazen and Helpman (1988), and Calvo and Drazen (1997) are mainly concerned with studying uncertainty about the timing of stabilization, we are interested in dealing with uncertainty about the exchange rate dynamics.

Fréchet-Weibull type. It is important to point out that incorporating an extreme value distribution of the Fréchet-Weibull type for the exchange-rate stochastic dynamics extends the work in Venegas-Martínez (2000) and (2006a).

In the framework of partial equilibrium, the proposed model will assume that contingent-claims markets for hedging against devaluation are unavailable. In this context and assuming logarithmic utility, which provides risk-averse agents, we shall examine the equilibrium dynamics of consumption and real wealth when a stabilization program is implemented and the size of devaluation is expected to follow an extreme value Fréchet distribution. We shall also study the effects on economic welfare and consumption of once-and-for-all changes in the parameters determining the expectations. The model is developed under the following two main assumptions: the revenue raised by seignorage is not rebated back to the agents and policy variables are stochastic. Finally, it is important to mention that the proposed model derives tractable closed-forms solutions that make much easier the understanding of the key issues in the analysis of temporary stabilization when the expected size of a possible devaluation is supposed to have an extreme value distribution.

This research is organized as follows. In the next section, we work out a one-good, cash-in-advance, stochastic model where a representative agent has expectations of devaluation driven by a mixed diffusion-jump process where the expected size of a possible devaluation is supposed to have an extreme value distribution. Through section 3, we solve the consumer's decision problem. In section 4, we undertake several policy experiments through comparative statics exercises. In section 5, we examine the impact of exogenous shock on economic welfare. In section 6, we study the dynamic behavior of wealth and consumption, and address a number of exchange-rate policy issues. Finally, in section 7, we present conclusions, acknowledge limitations, and make suggestions for further research.

■ *Structure of the economy*

In what follows, we set up the characteristics of the economy under study. The behavior of both the exchange rate and the available assets will be introduced.

Exchange rate dynamics

In order to derive solutions which are analytically tractable, the structure of the economy will be kept as simple as possible. Let us consider a small open economy[6] with a single infinitely lived consumer in a world with a single perishable consumption good. We assume that the good is freely traded, and its domestic price level, P_t, the purchasing power parity condition, namely

$$P_t = P_t^* e_t, \tag{1}$$

[6] We are mainly concern with small and open economies with a regime including a devaluation rate.

Where P_t^* is the foreign-currency price of the good in the rest of world, and e_t is the nominal exchange rate. Throughout the paper, we will assume, for the sake of simplicity, that P_t^* is equal to 1. We also assume that the exchange-rate initial value, e_0, is known and equal to 1.

In what follows, we will suppose that the ongoing uncertainty in the dynamics of the expected rate of devaluation, and therefore in the inflation rate, is generated by the following process geometric Brownian motion combined with Poisson jumps of a random sizes driven by extreme value distributions of the Fréchet-Weibull type:

(2) $$\frac{\mathrm{d}e_t}{e_t} = \frac{\mathrm{d}P_t}{P_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t + Z^+ \mathrm{d}N_t^* - Z^- \mathrm{d}N_t^-$$

where the trend $\mu \in \Re$ (a real number) is the expected (annualized) mean of the exchange rate, $\sigma > 0$ is the instantaneous (annualized) volatility of the exchange rate, $(W_t)_{t\geq 0}$ is a Brownian motion defined on a fixed probability space $\left(\Omega, F, \left(F_t^W\right)_{t\in[0,T]}, P_W\right)$ to model small movements in the exchange rate, and $\mathrm{d}N_t^+$ and $\mathrm{d}N_t^-$ are both Poisson processes with intensity parameters λ^+ and λ^- representing the number of upward and downward jumps, respectively, of the exchange rate per unit of time. We will assume that $\lambda^+ > \lambda^-$. The size of the upward and downward jumps satisfy, respectively,

$$Z^+ = \frac{1}{-\left(X^+\right)^{-\alpha^+} + 1} - 1, \quad X^+ > 0, \alpha^+ > 0,$$

$$Z^- = \frac{1}{-\left(-X^-\right)^{-\alpha^-} + 1} - 1, \quad X^- < 0, \alpha^- < 0,$$

$$X^+ = \frac{Y^+ - \mu^+}{\sigma^+}, \sigma^+, \mu^+ > 0, \; X^- = \frac{Y^- - \mu^-}{\sigma^-}, \; \sigma^- > 0, \mu^- < 0.$$

We also suppose that Y^+ is an extreme value Fréchet random variable with parameters α^+, μ^+ and $\sigma^+ > 0$, that is, Y^+ has cumulative distribution function given by

(3) $$F_{Y^+}(y) = \begin{cases} 0, & y < \mu^+, \\ \exp\left\{-\left(\dfrac{y-\mu^+}{\sigma^+}\right)^{-\alpha^+}\right\}, & y \geq \mu^+, \end{cases}$$

and Y^- is an extreme value Weibull random variable with parameters α^- and $\mu^- < 0$, $\sigma^- > 0$, that is, Y^- has cumulative distribution function defined by

$$(4) \qquad F_{Y^-}(y) = \begin{cases} \exp\left\{-\left(-\dfrac{y-\mu^-}{\sigma^-}\right)^{-\alpha^-}\right\}, & y < \mu^-, \\ 1, & y \geq \mu^-. \end{cases}$$

The corresponding densities satisfy

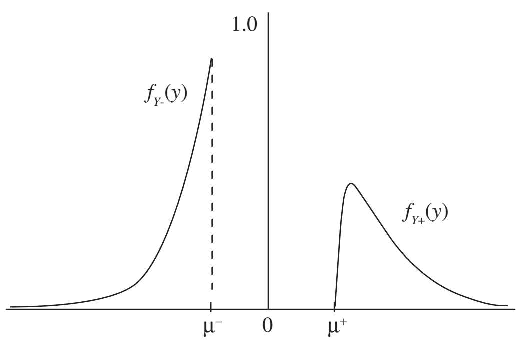
$$f_{Y^+}(y) = \frac{\alpha^+}{\sigma^+} F_{Y^+}(y)\left(\frac{y-\mu^+}{\sigma^+}\right)^{-(1+\alpha^+)}, \quad y \geq \mu^+$$

and

$$f_{Y^-}(y) = \frac{|\alpha^-|}{\sigma^-} F_{Y^-}(y)\left(-\frac{y-\mu^-}{\sigma^-}\right)^{-(1+\alpha^-)}, \quad y \leq \mu^-.$$

Figure 1 shows a Weibull and a Fréchet densities with given parameter values.

Figure 1
Weibull and Fréchet densities with $\sigma^+ = \sigma^- = \alpha^+ = |\alpha^-| = 1$.



Source: Prepared by authors.

Notice now that if $\sigma^+ > 2$, then

$$E[Y^+] = \mu^+ + \sigma^+\Gamma\left(1 - \frac{1}{\alpha^+}\right),$$

$$E[(Y^+)^2] = (\sigma^+)^2\Gamma\left(1 - \frac{2}{\alpha^+}\right) + 2\mu^+\sigma^+\Gamma\left(1 - \frac{1}{\alpha^+}\right) + (\mu^+)^2,$$

$$Var[Y^+] = (\sigma^+)^2\left[\Gamma\left(1 - \frac{2}{\alpha^+}\right) - \Gamma^2\left(1 - \frac{1}{\alpha^+}\right)\right].$$

Expressions for $\mathrm{E}\left[Y^-\right]$, $\mathrm{E}\left[(Y^-)^2\right]$, and $\mathrm{Var}\left[Y^-\right]$ are similar to that given above. On the other hand, since the number of expected devaluations (*i.e.*, upward jumps in the exchange rate) per unit of time follows a Poisson process $\mathrm{d}N_t^+$ with intensity λ^+, we have that

$$P_{N^+}\{\text{one unit jump during d}t\} = P_{N^+}\{\mathrm{d}N_t^+ = 1\} = \lambda^+ \mathrm{d}t$$

and

$$P_{N^+}\{\text{more than one unit jump during d}t\} = P_{N^+}\{\mathrm{d}N_t^+ > 1\} = o(dt)$$

so that

$$P_{N^+}\{\text{no jump during d}t\} = 1 - \lambda^+ \mathrm{d}t + o(dt)$$

where $o(\mathrm{d}t)/\mathrm{d}t \to 0$ as $\mathrm{d}t \to 0$. The process $\mathrm{d}N_t^-$ satisfies similar conditions to the above ones.

- *Assets available in the economy*

The representative consumer holds two real assets: real cash balances, $m_t = M_t/P_t$, where M_t is the nominal stock of money, and an international bond, b_t. The bond pays a constant real interest rate r (*i.e.*, it pays r units of the consumption good per unit of time). Thus, the consumer's real wealth, a_t, is defined by

(5) $$a_t = m_t + b_t,$$

where the initial real wealth a_0 is exogenously determined. Although this is a limitation of the proposed model, this will be used to think of the decision making process of the representative agent as a stochastic optimal control problem in continuous time. Furthermore, we assume that the rest of the world does not hold domestic currency. Consider a cash-in-advance constraint of the Clower type:

(6) $$m_t = \int_t^{t+\psi^{-1}} c_s \mathrm{d}s,$$

where c_t is consumption, and $\psi^{-1} > 0$ is the time that money must be held to finance consumption. Condition (6) is critical in linking exchange-rate dynamics with consumption. Observe that

$$m_t = \int_t^{t+\psi^{-1}} c_s ds \approx \psi^{-1} c_t + o(\psi^{-1}).$$

In the sequel, we will assume that the error $o(\psi^{-1})$ is negligible. In this case, devaluation acts as a stochastic tax on real cash balances.

The stochastic rate of return of holding real cash balances, $\mathrm{d}r_m$, is simply the percentage change in the price of money in terms of goods. By applying Itô's lemma to the inverse of the price level, with (2) as the equation driven the underlying process (see, for instance, Venegas-Martínez, 2008), we get

$$
\begin{aligned}
&\mathrm{d}\left(\frac{1}{P_t}\right)\\
&=\left[-\left(\frac{1}{P_t^2}\right)\mu P_t+\frac{1}{2}\left(\frac{2}{P_t^3}\right)\sigma^2 P_t^2\right]\mathrm{d}t-\left(\frac{1}{P_t^2}\right)\sigma P_t dW_t \qquad (7)\\
&+\left[\frac{-(X^+)^{-\alpha^+}+1}{P_t}-\frac{1}{P_t}\right]\mathrm{d}N_t^+-\left[\frac{-(-X^-)^{-\alpha^-}+1}{P_t}-\frac{1}{P_t}\right]\mathrm{d}N_t^-\\
&=\frac{1}{P_t}\left[(-\mu+\sigma^2)dt-\sigma dW_t-(X^+)^{-\alpha^+}dN_t^+ +(-X^-)^{-\alpha^-}dN_t^-\right].
\end{aligned}
$$

Hence, the stochastic rate of return of holding real cash balances, $\mathrm{d}R_m$, is given by

$$dR_m=(-\mu+\sigma^2)dt-\sigma dW_t-(X^+)^{-\alpha^+}dN_t^+ +(-X^-)^{-\alpha^-}dN_t^- \qquad (8)$$

■ *The consumer's choice problem*

The stochastic consumer's real wealth accumulation in terms of the portfolio shares $w_t=m_t/a_t$, $1-w_t=b_t/a_t$, and consumption, c_t, is given by

$$da_t=a_t w_t dR_m+a_t(1-w_t)rdt-c_t dt$$

with a_0 exogenously determined. Thus,

$$da_t=a_t\left[(r-\rho w_t)dt-w_t\sigma dW_t-w_t(X^+)^{-\alpha^+}dN_t^+ +w_t(-X^-)^{-\alpha^-}dN_t^-\right], \qquad (9)$$

where $\rho=\psi+r+\mu-\sigma^2$.

The von Neumann-Morgenstern utility at time $t=0$, V_0, of the competitive risk-averse consumer is assumed to have the time-separable form:

$$V_0 = E_0\left\{\int_0^{\infty} \log(c_t)e^{-rt}dt\right\}, \tag{10}$$

where E_0 is the conditional expectation on all available information at $t = 0$. To avoid unnecessary complex dynamics in consumption, we assume that the agent's subjective discount rate has been set equal to the constant real international interest rate, r. We consider the logarithmic utility function in order to derive closed-form solutions and make the analysis tractable.

First order conditions for a an interior solution

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing utility, with $\log(c_t) = \log(\psi a_t w_t)$, and assuming that Y^+ and Y^- are stochastically independent of dN_t^+ and dN_t^- , respectively, is given by (see, for instance, Venegas-Martínez, 2008).

$$\begin{aligned} \max_w H(w_t; a_t, t) = \max_w \{ &\log(\psi a_t w_t)e^{-rt} + I_a(a_t,t)a_t(r-\rho w_t) + I_t(a_t,t) \\ &+ \tfrac{1}{2} I_{aa}(a_t,t)a_t^2 w_t^2 \sigma^2 - \lambda^+ E[I(a_t(w_t(X^+)^{-\alpha^+} + 1),t) - I(a_t,t)] \\ &+ \lambda^+ E[I(a_t(w_t(X^+)^{-\alpha^+ + 1}),t) - I(a_t,t)]\} = 0 \end{aligned} \tag{11}$$

The first-order condition for an interior solution is:

$$H_{wt} = 0.$$

Given the exponential time discounting in (10), we postulate $I(a_t, t)$ in a time-separable form as:

$$I(a_t,t) = e^{-rt}[\beta_1 \log(a_t) + \beta_0], \tag{12}$$

where β_0 and β_1 are both to be determined from (11). In this case, we obtain

$$\begin{aligned} \max_w H(w_t; a_t, t) = \max_w \{ &\log(\psi a_t w_t) + \beta_1(r - \rho w_t) - r[\beta_1 \log(a_t) + \beta_0] \\ &- \tfrac{1}{2}\beta_1 w_t^2 \sigma^2 + \beta_1 L(w_t)\} = 0 \end{aligned} \tag{13}$$

Where

$$L(w_t) = -\lambda^+ E[\log(w_t(X^+)^{-\alpha^+} + 1)] + \lambda^- E[\log(w_t(-X^+)^{-\alpha^-} + 1)]. \tag{14}$$

Notice that the arguments in the logarithmic function above are both positive. We compute the first-order conditions by using

$$\frac{\partial}{\partial w_t} E\left[\log\left(w_t (X^+)^{-\alpha^+} + 1\right)\right] = E\left[\frac{\partial}{\partial w_t}\log\left(w_t (X^+)^{-\alpha^+} + 1\right)\right] = E\left[\frac{(X^+)^{-\alpha^+}}{w_t (X^+)^{-\alpha^+} + 1}\right].$$

In this case, we find that $w_t \equiv w$ is time-invariant and

$$\frac{1}{\beta_1 w} - \frac{\lambda^+}{w}\mathrm{E}\left[\frac{(X^+)^{-\alpha^+}}{(X^+)^{-\alpha^+} + w^{-1}}\right] + \frac{\lambda^-}{w}\mathrm{E}\left[\frac{(-X^-)^{-\alpha^-}}{(-X^-)^{-\alpha^-} + w^{-1}}\right] = \rho + w\sigma^2. \tag{15}$$

By defining the change of variable

$$\varsigma = \left(\frac{y-\mu^+}{\sigma^+}\right)^{-\alpha^+},$$

the first expectation can be written as

$$\begin{aligned} \mathrm{E}\left[\frac{(X^+)^{-\alpha^+}}{(X^+)^{-\alpha^+} + w^{-1}}\right] &= \int_0^\infty \frac{\left[(y-\mu^+)/\sigma^+\right]^{-\alpha^+}}{\left[(y-\mu^+)/\sigma^+\right]^{-\alpha^+} + w^{-1}} f_{Y^+}(y)\mathrm{d}y \\ &= \int_0^\infty \frac{\zeta}{\zeta + w^{-1}} e^{-\zeta}\mathrm{d}\zeta \\ &= \frac{1}{w} e^{1/w}\Gamma(-1, 1/w). \end{aligned} \tag{16}$$

where

$$\Gamma(-1,1/w) = \Gamma(0,1/w) + e^{-1/w}w,$$

and, for small w, in fact for $0 < w < 1$, we have the following approximation:

$$\Gamma(0,1/w) = \int_{1/w}^\infty \frac{e^{-u}}{u}\mathrm{d}u = we^{-1/w}(1 - w + o(w)), \tag{17}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function. Similarly,

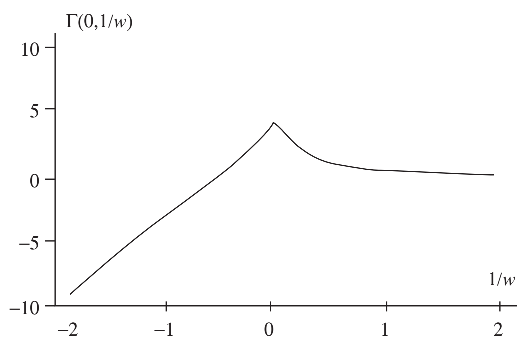
(18)
$$E\left[\frac{(X^+)^{-\alpha^-}}{(X^+)^{-\alpha^-}+w^{-1}}\right]=\int_0^\infty \frac{\left[-(y-\mu^-)/\sigma^-\right]^{-\alpha^-}}{\left[-(y-\mu^-)/\sigma^-\right]^{-\alpha^-}+w^{-1}} f_{Y^-}(y)\mathrm{d}y$$
$$=\frac{1}{w}e^{1/w}\Gamma(-1,1/w).$$

Hence, from (16), (17) and (18), the first order condition can be written as

(19)
$$\frac{1}{\beta_1 w}-\frac{\lambda^+}{w}\left(1-\frac{1}{w}e^{1/w}\Gamma(0,1/w)\right)=\rho+w\sigma^2.$$

where $\lambda=\lambda^+-\lambda^-$. Figure 2 shows the graph of the function $\Gamma(0,1/w)$.

Figure 2
Fuction $\Gamma(0,1/w)$



Source: Prepared by authors.

If we assume that $0<w<1$ and use (17), then we may leave out the error term and write the first order condition as

(20)
$$\frac{1}{\beta_1 w}-\lambda=\rho+w\sigma^2.$$

Once an optimal w is chosen, coefficients β_0 and β_1 are determined as follows

$$\log(\psi a_t w)+\beta_1(r-\rho w)-r[\beta_1\log(a_t)+\beta_0]-\frac{1}{2}\beta_1 w^2\sigma^2+\beta_1 L(w)=0$$

or

$$(1-r\beta_1)\log(a_t)-r\beta_0+r\beta_1+\log(\psi w)-\beta_1\left[\rho w+\frac{1}{2}w^2\sigma^2-L(w)\right]=0$$

which leads to

(21) $$\beta_1 = r^{-1}$$

and

$$\beta_0 = \frac{1}{r}\left[1 + \log(\psi w)\right] - \frac{1}{r^2}\left[\rho w + \frac{1}{2}w^2\sigma^2 - L(w)\right].$$

Thus,

$$\frac{r}{w} = \rho + \lambda + w\sigma^2,$$

which is a quadratic homogeneous equation with real solutions

(22) $$w^+ = \frac{-(\rho+\lambda)+\sqrt{(\rho+\lambda)^2+4\sigma^2 r}}{2\sigma^2} > 0$$

and

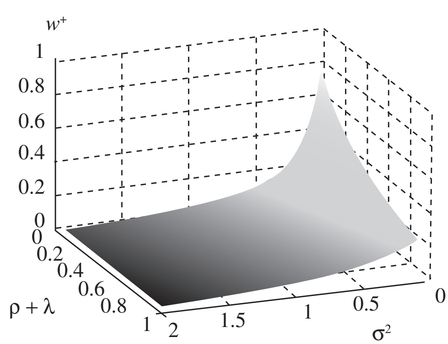
$$w^- = \frac{-(\rho+\lambda)-\sqrt{(\rho+\lambda)^2+4\sigma^2 r}}{2\sigma^2} < 0.$$

Notice that always $0 < \psi + \mu + \lambda$, and that *iff*

$$\sqrt{(\rho+\lambda)^2+4\sigma^2 r} < 2\sigma^2 + (\rho+\lambda),$$

iff $0 < w^+ < 1$. Figure 3 shows w^+ as a function of $\rho+\lambda$ and σ^2 with a constant value of r.

Figure 3
w^+ as a function of $\rho+\lambda$ and σ^2 with $r = 0.09$.



Source: Prepared by authors.

■ *Policy experiments and comparative statics*

In this section, we carry out some comparative statics experiments regarding the optimal share w^+. We will see the effects of changes in the mean expected rate of inflation μ, the instantaneous volatility of inflation and the total intensity parameter λ on w^+. By differentiating the first order condition, we get

$$\mathrm{d}\mu + (w^+ - 1)\mathrm{d}\sigma^2 - \mathrm{d}\lambda + A(w^+)\mathrm{d}w^+ = 0, \tag{23}$$

where

$$A(w^+) = \sigma^2 + \frac{r}{w^2}.$$

We are now in a position to derive our first result: a once-and-for-all increase in the rate of devaluation, which results in an increase in the future opportunity cost of purchasing goods, leads to a permanent decrease in the proportion of wealth devoted to future consumption. To see this, it is enough to use (23) to find that

$$\frac{\partial w^+}{\partial \mu} = -\frac{1}{A(w^+)} < 0. \tag{24}$$

Notice also that a once-and-for-all increase in the inverse of the variance of the diffusion component will produce a contrary effect to that of μ on w^+ since

$$\frac{\partial w^+}{\partial \sigma^2} = \frac{1 - w^+}{A(w^+)} > 0. \tag{25}$$

In other words, the consumer sets aside a larger proportion of wealth to maintain real monetary balances to finance consumption, in order to deal with a higher variance in consumption prices.

Another result is the response of the equilibrium share of real monetary balances, w^+ to once-and-for-all changes in the total-intensity parameter, λ. A once-and-for-all increase in the expected number of extreme devaluations per unit of time causes an increase in the future opportunity cost of purchasing goods. This, in turn, permanently decreases the proportion of wealth set aside for future consumption. From (11), we get

$$\frac{\partial w^+}{\partial \lambda} = -\frac{1}{A(w^+)} < 0. \tag{26}$$

■ *Impact on economic welfare*

We will now assess the effects of exogenous shocks on economic welfare. As usual, the welfare criterion, W, of the representative individual is the maximized utility starting from the initial real wealth, a_0. Therefore, welfare is given by

$$
\text{(27)} \quad \begin{aligned} W(\mu,\sigma^2;a_0) \equiv I(a_0,0) &= \frac{1}{r}\left[1+\log(a_0)+\log(\alpha^{-1}w^+)\right] \\ &-\frac{1}{r^2}\left[\rho w^+ + \frac{1}{2}(w^+)^2\sigma^2 - L(w^+)\right]. \end{aligned}
$$

Table 3 shows the impacts on welfare of once-and-for-all changes in the mean expected rate of devaluation, the inverse of volatility, the probability of devaluation, and the expected size from devaluation.

Table 3
Effects of policy changes on economic welfare

Increase in	Effect on welfare
ε	$-\frac{w}{r^2}<0$
σ^2	$-\frac{(w^+)}{r^2}<0$
λ	$-\frac{w^+}{r^2}<0$

In order to compute economic welfare W, we need to find, explicitly, $L(w^+)$. To this end, we use the same change of variable as in (16), so

$$
\text{(28)} \quad \begin{aligned} E\left[w^+(X^+)^{-\alpha^+}+1\right] &= \int_0^\infty \log(w^2\zeta+1)e^{-\zeta}d\zeta \\ &= e^{1/w}\Gamma(0,1/w). \end{aligned}
$$

It is important to point out that (16) can be also obtained by differentiating (28)

$$
\frac{\partial}{\partial w^+}e^{1/w}\Gamma(0,1/w^+) = -\frac{1}{(w^+)^2}e^{1/w^+}\Gamma(0,1/w^+)+e^{1/w^+}\left(\frac{\partial\Gamma(0,1/w^+)}{\partial(1/w^+)}\right)\left(\frac{\partial(1/w^+)}{\partial w^+}\right)
$$

$$
= -\frac{1}{(w^+)^2}e^{1/w^+}\Gamma(0,1/w^+)+e^{1/w^+}\left(-w^+e^{-1/w^+}\right)\left(-\frac{1}{(w^+)^2}\right)
$$

$$
= -\frac{1}{(w^+)^2}e^{1/w^+}\Gamma(0,1/w^+)+\frac{1}{w^+}.
$$

Notice also that since there is a differentiation process of $L(w^{+})$ in the first order condition, we may use now the approximation $\Gamma(0,1/w^{+}) = we^{-1/w}(1+O(w))$, thus

$$L(w^{+}) = -\lambda w^{+} = \int_0^{w^{+}} \lambda \mathrm{d}x.$$

■ *Wealth, consumption and dynamic implications*

We now derive the stochastic process that generates wealth when the optimal share is applied. After substituting the optimal share w^{+} into (9), we get

$$da_t = a_t\left[(\lambda + (w^{+})^2\sigma^2)dt - w^{+}\sigma dW_t - w^{+}(X^{+})^{-\alpha^{+}} dN_t^{+} + w^{+}(-X^{-})^{-\alpha^{-}} dN_t^{-}\right], \tag{29}$$

The solution to the above stochastic differential equation, conditional on a_0, is

$$a_t = a_0 e^{\delta_t}, \tag{30}$$

where

$$\delta_t = \eta_t - \gamma_t^{+} + \gamma_t^{-},\ \eta_t \sim N[F(w^{+})t, G(w^{+})t],\ \ \gamma_t^{+} = H^{+}(w^{+}, X^{+})N_t^{+},$$

$$\gamma_t^{-} = H^{-}(w^{+}, X^{-})N_t^{-},\ N_t^{+} \sim P(\lambda^{+}t),\ N_t^{-} \sim P(\lambda^{-}t),$$

$$F(w^{+}) = \lambda + \frac{(w^{+})^2\sigma^2}{2},\ G(w^{+}) = (w^{+})^2\sigma^2,\ H^{+}(w^{+}) = \log\left(w^{+}(X^{+})^{-\alpha^{+}} + 1\right),$$

and

$$H^{-}(w^{+}) = \log\left(w^{+}(-X^{-})^{-\alpha^{-}} + 1\right).$$

Notice that

$$E[\delta_t] = [F(w^{+}) - E[H^{+}(w^{+}, X^{+})]\lambda^{+} + E[H^{+}(w^{+}, X^{-})]\lambda^{-}]t$$

$$= \left[F(w^{+}) - \lambda e^{1/w}\Gamma(0, 1/w)\right]t \tag{31}$$

$$= \left[F(w^{+}) - \lambda w^{+}(1 - w^{+} + o(w^{+}))\right]t$$

In virtue of (6), the stochastic process for consumption, in (30), can be written as

$$c_t^{+} = \alpha^{-1} w^{+} a_0 e^{\delta_t}, \tag{32}$$

This indicates that, in the absence of contingent-claims markets, the devaluation risk has an effect on wealth via the uncertainty in δ_t, that is, uncertainty changes the opportunity set faced by the consumer. On the other hand, the devaluation risk also affects the composition of portfolio shares via its effects on w^+. Thus, a policy change will be accompanied by both wealth and substitution effects. We cannot determine the level of consumption in our stochastic framework. We can only compute the probability that, at a given time, a certain level of consumption occurs. Notice, however, that by Jensen's inequality, mean consumption satisfies:

$$E\left[c_t^+\right] \geq \alpha^{-1} w^* a_0 e^{E[\delta_t]}. \tag{32}$$

In contrast with our stochastic setting, c_t^+ shows a dynamic behavior, even if the rate of devaluation were expected to remain fixed forever. This is because δ_t is a time-varying, state-contingent variable. We may conclude that uncertainty is the clue to rationalize richer consumption dynamics that could not be obtained from deterministic models.

Consumption booms

Next, we will analyze a policy of the form:

$$\varepsilon_t = \begin{cases} \varepsilon_1 & \text{for } 0 \leq t \leq T, \\ \varepsilon_2 & \text{for } t > T, \end{cases} \tag{33}$$

where T is exogenously determined, and $\varepsilon_1 < \varepsilon_2$, as in Calvo (1986). Notice that in our stochastic setting, there is a lack of credibility even if we do not change the four parameters since agents always assign some probability to the event of currency devaluation. Let us examine the response of consumption to (33). From (32), we may write

$$\frac{c_{T+\Delta}^+}{c_T^+} = \frac{w_2^+}{w_1^+} h_T\left(\Delta; \varepsilon_1, \varepsilon_2\right),$$

where $h_T\left(\Delta; \varepsilon_1, \varepsilon_2\right) \equiv \exp\left\{-\left(\delta_T\left(\varepsilon_1\right) - \delta_{T+\Delta}\left(\varepsilon_1\right)\right)\right\}$ tends to 1 as $\Delta \to 0^+$ a.s. (almost surely). The limit means that although the stationary components of the parameters of η_t and γ_t are different before and after time T, such a difference becomes negligible when $\Delta \to 0^+$. Consequently,

$$\lim_{\Delta \to 0^+} c_{T+\Delta}^+ = c_T^+ \frac{w_2^+}{w_1^+} \quad \text{a.s.} \tag{34}$$

We also notice that $w_2^+ / w_1^+ < 1$, together with (34), imply $c_T^+ > \lim_{\Delta \to 0^+} c_{T+\Delta}^+$ a.s., indicating a jump (boom) in consumption at time T. If ε were to be constant forever, i.e., if $\varepsilon_t = \varepsilon_2$ for all $t \geq 0$, then we would have

(35) $$c^{+}_{T+\Delta} = c^{+}_{t} h_t(\Delta; \varepsilon_1, \varepsilon_2).$$

On the right-hand side of (35), the factor $h_t(\Delta; \varepsilon_1, \varepsilon_2) \to 1$ as $\Delta \to 0^+$ a.s. Hence, consumption would be continuous a.s. for all t. If the plan is expected to be temporary, then $c^{+}_{T} > \lim_{\Delta \to 0^+} c^{+}_{T+\Delta}$ a.s., indicating a jump in consumption at T, as we have shown above.

■ *Conclusions*

The "credibility literature" has by now exhausted a class of deterministic models aimed at explaining consumption dynamics. Most of the existing models ignore uncertainty providing a very elaborate economic interpretation of why uncertainty needs to be considered. After all, what produces expected temporariness is uncertainty. We have presented a stochastic model of exchange-rate-based stabilization with imperfect credibility where agents have expectations of devaluation driven by a mixed diffusion-jump process and the expected size of a possible devaluation is supposed to have an extreme value distribution of the Fréchet-Weibull type. An important feature of our formulation is that there is a lack of credibility even if we do not change the parameters determining the expectations of devaluation. By using a logarithmic utility, we have derived closed-form solutions to examine the dynamic implications of uncertainty. These explicit solutions have made much easier the understanding of the key issues of temporary programs.

Our stochastic framework, in which a Poisson process drives the expectations of devaluation and the expected size of a possible devaluation is supposed to have an extreme value distribution of the Fréchet-Weibull type, provides new elements to carry out comparative statics experiments and empirical research on some observed regularities in temporary stabilization that still need to be explained.

The broad message of this paper, although only demonstrated for a particular case of utility index, is that public expectations and consumption dynamics generated by exchange-rate-based stabilization plans are linked through fragile relationships. Therefore, policymakers should consider these elements with great caution when devising a corrective devaluation, especially if contingent-claims markets are absent.

It is worthwhile mentioning that the results obtained strongly depend on the assumption of logarithmic utility, which is a limit case of the family of constant relative risk aversion utility functions. The extension of our stochastic analysis to such a family does not provide closed-form solutions, and results might be only obtained via numerical methods. Needless to say, additional research is required in this direction.

The model can be obviously extended in several ways, for instance, more research should be undertaken in adding both non tradable and durable goods; this, of course, will provide more realistic assumptions. This extension will lead to more complex transitional dynamics, but results will certainly be richer.

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