Trade and Foreign Direct Investment Linkages: FDI versus Imports

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Resumen: Desarrollamos un modelo donde el gobierno del país huésped intenta atraer IED en presencia de competidores foráneos que exportan al país huésped un bien imperfectamente sustituto. El gobierno del país huésped es un agente maximizador de bienestar con dos estrategias de política impositiva: un impuesto por unidad de producto de las empresas foráneas, y una tarifa por unidad de producto sobre las importaciones. Mostramos que una tarifa positiva y un impuesto negativo al producto será el óptimo. Sin embargo, cuando ambas políticas impositivas se determinan simultáneamente, la política óptima es subsidiar a la IED. También, cuando las políticas impositivas son aplicadas uniformemente (como un impuesto al consumo) el impuesto óptimo es negativo.

Abstract: We develop a model where the host country government attempts to attract FDI in the presence of foreign competitors that export imperfectly substitute goods to the host country. The host country government is a welfare maximising agent with two available tax policies: a per unit output tax on foreign firms, and a per unit tariff on imports. We show that a positive tariff and a negative output tax will be optimal. However, when the policies are determined simultaneously the optimal combination is to have no tariff and to subsidize FDI. Also, when the governments tax policies are applied uniformly (as a consumption tax) the optimal tax is negative.

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Introduction

Over the period 1991-99, 94 percent of the changes regulating foreign investment created a more favorable framework for Foreign Direct Investment (FDI). According to UNCTAD, gross product associated with international production and foreign affiliate sales worldwide increased faster than global GDP and global exports. Thus, international production is seen as more important than international trade in delivering goods and services to foreign markets (UNCTAD 1999, 2000; and 2001).

Most developing countries today are trying to attract FDI. In general, those countries see FDI as a way of compensating for existing deficiencies in the local market, such as inferior production technology and management skills or limited access to world markets. As a consequence of these shortcomings, many developing countries are unable to produce certain essential goods. These economies may lack the technological knowledge for those particular goods or local capital markets may not be sufficiently developed to meet the capital requirements for the production of large investment goods. Moreover, the foreign capital attracted to these economies may not be enough to purchase the intermediate goods that are not available domestically. Under such circumstances, it may not be possible to produce certain goods locally. In that case the goods have to be imported from abroad.

Another way to overcome these deficiencies is to attract foreign firms. Various policies (such as taxation/subsidisation) may be used to encourage FDI to flow into the host country. Globally speaking, there now exists fierce competition amongst many countries interested in attracting FDI. Hence, the relationship between competition for FDI and the actual flow of foreign investment has become an important issue for both politicians and researchers. In this paper, we examine the effects of discriminatory and uniform tax policies on the number of foreign firms willing to invest in and on the number of exporting firms willing to export to the host country.

Apart from overcoming those deficiencies mentioned above, an economy may also benefit from FDI through a reduction in unemployment. In fact, to make our analysis more general we assume that there is unemployment in the host country as in Brander and Spencer (1987).

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4. See Balasubramanyam (2001) for a broad list of the determinants of FDI.
5. Brander and Spencer (1987) explicitly address the effects of FDI on the level of employment by constructing a model in which a foreign firm has to decide whether to invest or export to a potential host country where there is unemployment.
The literature on FDI is vast. However, those who analyse the effect of FDI on host country welfare are more close to our paper as we follow a similar path. Lahiri and Ono (1998a, 1998b), also analyse the effects of FDI on host country welfare. While the available policies to the host government are local content requirements and a tax (or subsidy) on profits in the first paper, lump sum profit subsidies are used in the latter. The increased use of subsidies to attract FDI is also studied in Haaparanta (1996) where investment is considered divisible so that firms can invest simultaneously in many countries. Haufler and Wooton (1999) discuss the case of competition between two countries to attract the investment of a single foreign firm. They consider a model of a region consisting a big country and a small country. Two alternative policy scenarios are considered. In the first case, governments use lump sum taxes (subsidies), and in the second they use either tariffs or consumption taxes. As a result of the size difference, the big country always will win the tax/subsidy war regardless of policy scenario. In a similar work, Haaland and Wooton (1999) show that a host country benefits from FDI through increased employment as well as through the demand of FDI for domestically produced intermediate goods.

When analyzing the effects of FDI on the welfare of host country economies, it is always assumed in the literature that there are also domestic firms in the sector that FDI is active. In this paper, we depart from this approach by assuming the absence of domestic competition. Today, many countries are not able to produce domestically, such as cars, camcorders etc. Instead, they either import those goods or let the multinational corporations (MNCs) to produce those within the country. In such countries, it is important for the government to encourage FDI not only for benefits such as employment and consumer surplus but also to substitute imported goods. Surprisingly, the literature on FDI ignored this possibility. Here, assuming the lack of domestic production, we develop a model where the host country government attempts to attract FDI in the presence of foreign competitors that export imperfectly substitute goods to the host country.

In this paper, we develop a partial equilibrium model of an oligopolistic industry in which an endogenous number of foreign-owned firms and foreign firms located abroad compete in the market for two differentiated commodities in a host country. The number of firms in the economy is a function of tax policy. Within this framework, we examine the effects of discriminatory and uniform tax policies on the number of foreign firms willing to invest in and on the number of exporting firms willing to export to the host country, as well as the effect on employ-
ment. Finally, we analyse the role of the degree of differentiation on products.

In the next section, we detail the main features of the basic model. In section 3, we examine the effects of policies on the number of both groups of firms. The optimal policies are derived in sections 4 and 5. Finally, we conclude in section 6.

Model

An endogenous number $(n^a)$ of identical firms, export goods to a host country market. These firms operate outside the host country and do not shift operations to the host country. Another group of firms, coming from the rest of the world and whose number is endogenously determined $(n^b)$, invest in the host country. Both $n^a$ and $n^b$ firms compete in an oligopolistic industry and produce two imperfectly substitutable goods. We assume that there are no domestic firms. The host country demand for the goods produced by the two types of firms are given by:

\[
D^a = n^a x^a, \\
D^b = n^b x^b,
\]

where $x^a$ and $x^b$ are the per firm output of the two types of firms respectively, $D^a$ and $D^b$ stand for the total domestic demand for the goods produced by exporting firms and FDI. The inverse demand functions for these commodities are given by: 7

\[
\begin{align*}
(3) & \quad p^a = \alpha^a \beta^a D^a - \gamma D^b, \\
(4) & \quad p^b = \alpha^b \beta^b D^b - \gamma D^a,
\end{align*}
\]

where $\gamma$ represents the degree of differentiation between the two goods such that $\beta^a > \gamma, \beta^b > \gamma$. If $\gamma = 0$, the commodities are completely differentiated; if $\gamma = \beta^a = \beta^b = \beta$ and $\alpha^a = \alpha^b = \alpha$, the commodities are perfectly homogeneous.8 The price of commodity $x^a$ is denoted by $p^a$,

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6. Implicitly, we assume that the firms’ decision on the mode of entry is made exogenously.
7. The inverse demand functions are derived from the quasi-linear utility function $U(D^a,D^b,y) = \alpha D^a + \alpha D^b - [\beta D^a + \beta D^b + 2\gamma D^a D^b + y] + \gamma D^a D^b + 2 + y$, where $y$ is consumption of the numeraire good.
8. We assume that products are differentiated according to their place of production. There are many evidence that fit into this assumption. For example, in many developing
and that of good \( x^e \) by \( p^e \). The marginal costs of the two types of firms, \( (c^a \) and \( c^b \) are assumed to be constant and therefore equal to average variable cost.\(^9\)

The profits \( \pi^a \) and \( \pi^b \) are given by:

\[
\begin{align*}
\pi^a &= (p^a - c^a - t^a) x^a - F, \\
\pi^b &= (p^b - c^b - t^b) x^b,
\end{align*}
\]

where \( t^a \) denotes a unit tariff imposed by the host country government on the exporting firms, and \( t^b \) denotes a per-unit output tax imposed by the host country government that is paid by the FDI-type firms.\(^10\) These tax policies allow the host country government to affect the number of exporting and investing firms. We assume that the host country is small in the market for FDI. Therefore, the foreign firms \( (n^e) \) move into (out of) the host country if the profit they make there, \( \pi^b \), is larger (smaller) than a reservation profit \( \pi^b \), that they could make in the rest of the world. Similarly, \( n^a \) firms will export (not export) to the host country if the profit they make, \( \pi^a \), is larger (smaller) than the fixed costs, \( F \), of production. The equilibrium conditions will then be:

\[
\begin{align*}
\pi^a &= F, \\
\pi^b &= \pi^b
\end{align*}
\]

The firms are assumed to behave in a Cournot-Nash fashion. Hence, profit maximisation yields:

\[
\begin{align*}
\beta x^a &= p^a - c^a - t^a \\
\beta x^b &= p^b - c^b - t^b
\end{align*}
\]

---

9. There is one factor of production whose price is determined in the competitive sector. Hence, marginal costs in the other sector can be taken as constant.
10. It is implicitly assumed that the cost of transportation for the exporting firms is negligible.
Given \( n^a \) and \( n^b \), profit maximising equilibrium output for both types of firms can be found by substituting (3) and (4) in (9) and (10); b:

\[
\begin{align*}
\frac{x^a}{\theta_1} &= \frac{\beta^a(1 + n^a)(\alpha^a - c^a - \ell^a) - \gamma n^b(\alpha^a - c^b - \ell^b)}{\theta_1} \\
\frac{x^b}{\theta_1} &= \frac{\beta^b(1 + n^b)(\alpha^b - c^b - \ell^b) - \gamma n^a(\alpha^a - c^a - \ell^a)}{\theta_1}
\end{align*}
\]

where \( \theta_1 = \beta^a \beta^b (1 + n^a)(1 + n^b) \) \( \gamma n^a n^b > 0 \). By using equations (3), (9) and (5) for optimal \( \pi^a \); and (4), (10) and (6) for \( \pi^b \), we find optimal profits as:

\[
\begin{align*}
\pi^a &= \beta^a x^a - F \\
\pi^b &= \beta^b x^b
\end{align*}
\]

Using this model specification, we obtain the following closed form solutions for output levels and the number of firms.\(^{11}\)

\[
\begin{align*}
\frac{\sqrt{\pi}}{\beta^a} &= \frac{\beta^a(1 + n^a)(\alpha^a - c^a - \ell^a) - \gamma n^b(\alpha^a - c^b - \ell^b)}{\theta_1} \\
\frac{\sqrt{\pi^b}}{\beta^b} &= \frac{\beta^b(1 + n^b)(\alpha^b - c^b - \ell^b) - \gamma n^a(\alpha^a - c^a - \ell^a)}{\theta_1}
\end{align*}
\]

\[
\begin{align*}
n^a &= \frac{\sqrt{\beta^a \pi^b}}{\sqrt{\beta^a \pi^b}} \left[ \beta^a(\alpha^a - c^a - \ell^a) - \gamma(\alpha^a - c^a - \ell^a) \right] - \beta^a \sqrt{\beta^a \pi^b} + \gamma \sqrt{\beta^a \pi^b} \right] \\
n^b &= \frac{\sqrt{\beta^b \pi^b}}{\sqrt{\beta^b \pi^b}} \left[ \beta^b(\alpha^b - c^b - \ell^b) - \gamma(\alpha^b - c^b - \ell^b) \right] - \beta^b \sqrt{\beta^b \pi^b} + \gamma \sqrt{\beta^b \pi^b} \right] \\
\end{align*}
\]

We assume, as in Brander and Spencer (1987), that there is unemployment in the host country. The variable input costs of FDI in the host country are taken to be the income of the nationals of that country. Therefore, increasing the number of firms in the host country will in-

\[11\] Equations (15) and (16) provide a system of equations. Solving for the values of \( n^a \) and \( n^b \) provides two pairs of roots. The feasible pair is given by (17) and (18). A second pair is ruled out as it provides infeasible results. See appendix for details.
crease employment and income through job creation. We also assume that the foreign investors are allowed to transfer all profits to their home country. Hence, the welfare \((W)\) of a representative consumer in the host country can be written as:

\[
W = t^aD^a + t^bD^b + c^bD^b + CS
\]  

where the first two terms are the tariff and tax revenues; the third term is the effect of employment, and the last term denotes consumers’ surplus.

With this equation we complete the model specification and turn to its analysis in the following sections.

\section*{Comparative Statics}

Up to now, we have taken the tariff \(t^a\) and output tax \(t^b\) as given. However, the host country government can use these instruments to affect the number of both types of firms. In order to see how tax policies may affect the number of firms, we need to define \(n^a\) and \(n^b\) in terms of policy parameters \(t^a\) and \(t^b\). Totally differentiating \(n^a\) and \(n^b\), we find:

\[
dn^a = \frac{\sqrt{\beta^a \beta^b}}{\sqrt{\pi^a (\beta^a \beta^b - \gamma^2)}} dt^a + \frac{\sqrt{\beta^a \gamma}}{\sqrt{\pi^a (\beta^a \beta^b - \gamma^2)}} dt^b
\]

\[
dn^b = \frac{\sqrt{\beta^b \gamma}}{\sqrt{\pi^b (\beta^b \beta^a - \gamma^2)}} dt^a - \frac{\sqrt{\beta^a \beta^b}}{\sqrt{\pi^b (\beta^b \beta^a - \gamma^2)}} dt^b
\]

Therefore, we have:

\[
\frac{dn^a}{dt^a} < 0 \quad \frac{dn^a}{dt^b} > 0 \quad \frac{dn^b}{dt^a} > 0 \quad \frac{dn^b}{dt^b} < 0
\]

Increasing the output tax \(t^b\) will reduce the number of firms \((n^b)\), i.e., less FDI will move into the host country. On the other hand, with higher costs for rival firms (due to an increase in \(t^a\)), investment becomes more attractive as incoming firms are more likely to have less competition. Similar intuition holds for the exporting firms \(n^a\).

\section*{Welfare and Discriminatory Policies}

In this section, we consider the cases when the government of host country applies discriminatory tax policies. In particular, we analyse (i)
the effects of a tax on imports but not on FDI production; (ii) a tax on FDI production but not on imports; (iii) and the case where both goods are simultaneously taxed, but at different ratios.

It is a well known fact that:

\[ dCS = -D^a dp^a - D^b dp^b \]  

In order to analyse optimal tax policies, we totally differentiate the welfare equation (19) (see appendix for the details):

\[ dW = \frac{(r^b + c^b)\gamma - \beta^b r^a}{(\beta^a \beta^b - \gamma^2)} \frac{\gamma - \beta^b (r^b + c^b)}{(\beta^b - \gamma^2)} dt^a + \frac{\gamma - \beta^b (r^b + c^b)}{(\beta^a \beta^b - \gamma^2)} dt^b \]

**Import Tariffs**

When there are no initial taxes or tariffs \((t^a = t^b = 0)\), the effect of an infinitesimal increase in tariff \(t\) on welfare can be found from (23) as:

\[ \left. \frac{\partial W}{\partial t^a} \right|_{r^a = t^a = 0} = \frac{c^b \gamma}{\beta^b - \gamma^2} \]

An infinitesimal increase in the tariff rate will have a positive effect on welfare. This is due to the employment effect, with employment and incomes increasing with a greater number of FDI.\(^{12}\) There are two additional effects on welfare of increasing the tariff rate. First an increase in price will decrease the consumer surplus. Second, the increase in tariff will increase the tariff revenue. However, these two effects exactly offset one another. With higher tariffs there will be less exporters selling to the host country, which creates a market-share advantage for the foreign investors. Therefore, more FDI will move into the host country. Hence, it is in the interest of the host country government to use the tariff instrument as a tool to attract more foreign firms and thereby to benefit from a higher level of employment. In this case, the optimal tariff is positive.\(^{13}\)

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\(^{12}\) The income of employed factors is defined as \(c^b D^b\) in (19). Let \(E = c^b D^b\). Differentiating this expression yields \(\beta^b (r^b + c^b) dE = c^b (\gamma r^b - \beta^b r^a)\), which reveals the employment effect of public policies on welfare.

\(^{13}\) From (23) one can determine that the optimal tariff is positive and the second order condition holds as \(W_{t^a t^a} = -\beta^b (\beta^b - \gamma^2) < 0\).
It is clear from (24) that as the degree of differentiation, \( \gamma \), tends to zero (i.e., when the commodities are almost perfectly differentiated), the effect of tariff on welfare also approaches zero. Since the goods are almost perfectly different, a tariff on imports will not encourage inflows of FDI to flow in any more. The optimal tariff is zero in this case.

At the other extreme, i.e., when the commodities are almost homogenous (\( \gamma = \beta = \beta' = \beta^b \) and \( \alpha = \alpha' = \alpha^b \), (when the number of both types of firms is endogenous) we find corner solutions where \( n^a \) or \( n^b \) is equal to zero. That is, one group of firms -the ones with higher marginal costs-will be forced out of the market. Rewriting the above results formally,

**Proposition 1.** In the absence of any initial policy towards FDI, the optimal tariff level on firms exporting to the host country is positive. The tariff approaches zero if the goods are almost perfectly differentiated (i.e., \( \gamma \approx 0 \)).

**Production Tax for FDI**

We shall now analyse the effect of an infinitesimal increase in production tax \( t^b \) on welfare when taxes and tariffs are initially absent \( (t^a = t^b = 0) \). Using (23):

\[
(25) \quad \left( \beta^a - \beta' - \gamma \right) \frac{\partial W}{\partial t^b} \bigg|_{t^a, t^b} \beta^b c^b
\]

Taxing foreign firms reduces the welfare by lowering employment in the host country. A small increase in the per-unit output tax on FDI reduces the profits of foreign firms, discouraging entry to the host country and lowering the level of employment. The term on the right hand side of (25) represents the negative employment effect of an infinitesimal increase in the output tax. Thus the optimal policy is to provide production subsidy for FDI.\(^{15}\) An infinitesimal increase in the output tax has two additional effects on welfare, lowering the consumer surplus and creating tax revenue for the government. However, similar as the previous case, these two effects exactly offset one another.

Equation (25) also reveals that the optimal policy remains a subsidy to FDI even when the commodities are almost perfectly differentiated. Subsidising FDI increases the employment by attracting more foreign investors. Formally,

\[14. \text{This result applies to all cases when the goods are almost perfect substitutes and the number of both groups of firms is endogenous.}\]

\[15. \text{The second order condition follows from (23) and holds as } W_{t^b} = -\beta^b(\beta' - \gamma) < 0. \]
Proposition 2. In the absence of any initial policy towards the firms exporting to the host country, the optimal level of output tax on FDI is negative.

Simultaneous Policies

Next, we shall find the optimal taxes when the government applies simultaneous but discriminatory tax policies. Setting the coefficients of $dta$ and $dtb$ in (23) equal to zero and solving them simultaneously for optimal tariff, $t^a$, and optimal output tax, $t^b$, we get the following results.

\begin{align*}
    (26) & \quad t^a = 0 \\
    (27) & \quad t^b = -c^b
\end{align*}

This result suggests that when the discriminatory tax policies are applied simultaneously the optimal combination is to have no import tariff and to subsidise FDI.\(^{16}\) The country essentially maximises the intensity of potential competition by setting the tariff equal to zero and then tilts the playing field in favour of the employment-generating firm by offering a unit subsidy equal to the unit employment benefit. Since the optimal subsidy is equal to the per unit employment benefit, the host country does not obtain any net employment benefit. Thus, there is no reason to favour FDI firms at all and it is optimal to set a zero tariff. Then, why is the subsidy set equal to minus marginal cost? Because this allows the host country to enjoy a consumer surplus equal to the whole area under the demand curve at no cost since the subsidy cost and employment benefit exactly offset each other. Since, with free entry the country can never do better than capture the whole area under the demand curve, this must be optimal. This result can be stated formally,

Proposition 3. When discriminatory tax policies for exporters and FDI are simultaneously determined, and the number of both types of firms is endogenous

(a) the optimal tariff is zero,

(b) the optimal output tax is negative.

\(^{16}\) Obviously, these results are optimal only if second order conditions are satisfied. That is, $W_{\beta p} < 0$ and $W_{\alpha p} < 0$ must hold. Furthermore, $[W_{\alpha p} W_{\beta p} W_{\alpha p} W_{\beta p} > 0]$ has to be satisfied. These conditions do hold in this case and the welfare function is concave in taxes and tariffs. See appendix for the proof.
Uniform Policy: A Consumption Tax

We now analyse the case where the taxes are uniform. Since the entire demand is coming from the host country, the output tax and tariff can be thought as consumption taxes. In particular, we analyse the case where the government applies nondiscriminatory consumption taxes. By equating $r = t = T$ in (23), we can examine the effect of an infinitesimal change in the uniform tax on welfare in the absence of any initial tax policy. Thus, we have:

\[
(28) \quad (\beta^a \beta^b - \gamma) \frac{dW}{dT} \bigg|_{r=T=0} = -c^b(\beta^a - \gamma)
\]

which is negative since $\beta^a > \gamma$. That is, an increase in the uniform tax has a negative effect on welfare through lost employment. Levying a positive tax uniformly will increase unemployment due to decreasing FDI, and increase prices. Hence, the optimal policy in this case is to subsidise both types of firms. This is an interesting result in the sense that the optimal policy for the host country government is to subsidise not only the FDI but also the firms that export to the country.\(^{17}\)

Equation (28) also reveals that the effect on welfare remains negative when the commodities are almost perfectly differentiated. This follows from the fact that a positive tax reduces the profits of FDI and makes them leave the host country, reducing employment. Formally,

**Proposition 4.** When the government’s tax policies are applied uniformly to both FDI and the exporting firms the optimal uniform tax is negative.

Conclusion

In this paper, we investigate the effects of foreign direct investment (FDI) on a host economy under tax policies that can be used to attract foreign firms. When examining the movements of FDI, the number of firms producing in and exporting to a host economy are taken as endogenous. The host country government, which is small in the market for FDI, is a welfare maximising agent with two available tax policy instruments; a per-unit output tax and a tariff modelled as a per-

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\(^{17}\) Once again, this result is optimal only if second order condition is satisfied. The condition holds and we have a welfare function concave in the uniform tax as $W_{TT} = -(\beta^a + 2\gamma)(\beta^b - \gamma) < 0$. 

unit import tax. Both groups of firms compete in the market for two imperfectly substitute goods, with the effect of changing the degree of substitutability also explored in this work. Two main characteristics of the model are the two gaps that exist in the host country: the existence of unemployment and the disability of the host country in domestically producing the two goods. Finally, we further investigate the effect of changing the assumptions about the endogeneity of the number of both types of firms.

We use this specification to analyse the effects of government policies on welfare. In particular, we examine the effect of both discriminatory and uniform tax policies on the level of employment and welfare. We find that the effect of a tariff on the level of employment is always positive as long as the commodities are not completely differentiated. A positive tariff works as an incentive for FDI to enter the host country by decreasing the number of exporting rivals.

An output tax is found to always have a negative effect on employment. The optimal output tax is negative given the number of foreign firms is endogenous. Finally, when the policies are applied uniformly, the level of employment always decreases under a positive uniform tax. When there are endogenous number of firms of both types the optimal uniform tax is unambiguously negative, implying a uniform subsidy.

\section*{Appendix A: Mathematical Details}

Solving (15) and (16) respectively for the number of firms we get two reaction functions \( b \)

\begin{equation}
(A. 1) \quad n^e = \frac{\sqrt{\beta^a \beta^b (1 + n^e)(\alpha^e - c^e - r^e) - \gamma n^e (\alpha^b - c^b - r^b)}}{\theta_3 \sqrt{F}} - \frac{\sqrt{F \beta^a \beta^b (1 + n^e)}}{\theta_3 \sqrt{F}}
\end{equation}

\begin{equation}
(A. 2) \quad n^b = \frac{\sqrt{\beta^b \beta^a (1 + n^b)(\alpha^b - c^b - r^b) - \gamma n^b (\alpha^a - c^a - r^a)}}{\theta_3 \sqrt{F}} - \frac{\sqrt{\pi^a \beta^b \beta^a (1 + n^b)}}{\theta_3 \sqrt{F}}
\end{equation}

where \( \theta_3 = \beta^a \beta^b (1 + n^e) - \gamma n^e > 0 \) and \( \theta_4 = \beta^b \beta^a (1 + n^b) - \gamma n^b > 0. \)
By solving (A.1) and (A.2) we find two pair of roots. The first one is given by (17) and (18). The second pair of roots is defined as follows

\[(A.3) \quad n^a = \frac{\beta^A}{(\gamma A - \beta^B)} \quad \text{and} \quad n^b = \frac{\beta^B}{(\gamma B - \beta^A)}\]

where \(A = (\alpha^a - c^a - r^a) > 0\) and \(B = (\alpha^b - c^b - r^b) > 0\). For feasible levels of output we need the following conditions on \(n^a\) and \(n^b\).

\[x^a > 0 \text{ if } n^a > \frac{\beta^B}{(\gamma A - \beta^B)} \quad x^b > 0 \text{ if } n^b > \frac{\beta^A}{(\gamma B - \beta^A)}\]

Therefore, we rule out the second set of roots seen in (A.3) as they do not match with the conditions that satisfy feasible output levels.

Totally differentiating (19) we obtain:

\[(A.4) \quad dW = t^a dD^a + (\ell^a + c^a) dD^a + D^a d\ell^a + D^a d\ell^a + dCS\]

where:

\[(A.5) \quad dD^a = x^a dn^a + n^a dx^a\]
\[(A.6) \quad dD^b = x^b dn^b + n^b dx^b\]

which is found by totally differentiating (1) and (2). Totally differentiating equations (3) and (4) we get:

\[(A.7) \quad dp^a = - \beta^a dD^a - \gamma D^a\]
\[(A.8) \quad dp^b = - \beta^b dD^b - \gamma D^b\]

Finally, by substituting (A.7) and (A.8) in (22), and using (A.5) and (A.6) we get

\[(A.9) \quad dCS = (\beta^a D^a + \gamma D^a)(x^a dn^a + n^a dx^a) + (\beta^b D^b + \gamma D^b)(x^a dn^b + n^b dx^b)\]

where \(dx^a = 0\) (both in (A.5) and (A.9)) and \(dx^b = 0\) (both in (A.6) and (A.9)) at equilibrium when the number of both types of firms is endogenous. This result can be found by totally differentiating equa-
tions (15) and (16). Next we substitute (20) and (21) in (A. 9), and all the results in (A. 4). Finally, substituting from (15) to (18) and after collecting the common terms we obtain equation (23).

When both types of firms are endogenous in number, equations (26) and (27) are optimal only if the second order conditions are satisfied. That is, $W_{\text{tata}} < 0$ and $W_{\text{tbtb}} < 0$ must hold. Furthermore, $[W_{\rho \rho} W_{\rho \rho} - W_{\rho \rho} W_{\rho \rho} > 0]$ has to be satisfied.

Using (23) gives:

(A. 10) $W_{\phi \phi} \frac{-\beta}{(\beta \beta - \gamma)} < 0$

(A. 11) $W_{\phi \phi} \frac{-\beta}{(\beta \beta - \gamma)} < 0$

(A. 12) $W_{\phi \phi} W_{\phi \phi} \frac{\gamma}{(\beta \beta - \gamma)} < 0$

(A. 13) $W_{\phi \phi} W_{\phi \phi} W_{\phi \phi} W_{\phi \phi} \frac{1}{(\beta \beta - \gamma)} > 0$

References


